TEACHING AND LEARNING MATHEMATICS HUMANISTICALLY

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Usually, professor of mathematics shows a few examples, and assigns students some similar exercises for drill. As a result many students come to view that mathematics is useless in real life, boring, and incomprehensible. Teaching mathematics humanistically demands creativity of both the professor and student. This theme emphasizes the meaning of the emotional climate of the activity of learning mathematics. Lots of promising ideas float around: historical background showing mathematics as a human discipline, interesting problems and open-ended questions, supreme beauty in mathematics, working in groups and teaching by projects, using of computer algebra systems, mathematics application in business, economics, art, and architecture. Which way should we go?

HISTORICAL BACKGROUND

How can we design mathematical courses that instill in students a sense of excitement about making connections in mathematics? One approach is to introduce a discussion via topics familiar to students, and to expand the dialogue with historical references and activities that lead to a discovery of new directions or themes in mathematics.

An example of a surprising fact in mathematics can be found in connection with regular polyhedrons. Polyhedrons fascinate many people. Already Pythagoras in the 500s BC knew that there can exist only five types of regular polyhedrons, with respectively 4 (tetrahedron), 6 (cube), 8 (octahedron), 12 (dodecahedron) and 20 (icosahedron) polygonal lateral faces. The regular polyhedrons were associated with the four elements in Greek philosophy: fire (tetrahedron), air (octahedron), earth (cube), water (icosahedron). The dodecahedron was associated with an image of the universe itself.

It is not difficult to speculate as to whether there exist more than the five regular polyhedrons. No kinds of experiments are sufficient to get a final answer to this question. It can only be settled by a mathematical proof. A proof can be based on the theorem that the alternating sum of the number of vertices, edges and polygonal faces in the surface of a convex polyhedron equals 2, stated by Euler.

PROBLEM SOLVING AND GROUP PROJECTS

First of all, a problem is a non-routine task which is being encountered by the students for the very first time and, therefore, there is no obvious algorithm for the student to use. There are two kinds of mathematical problems: problems to find the unknowns and problems to prove which comprise a conjecture.

After this, a problem is a task which has certain open questions that challenge the student emotionally and intellectually. A problem is relative to the students involved, i.e. what is a problem for one student may be an exercise for another. For example the task to solve the equation $x^8 - 1 = 0$ may be may be a problem for a schooler but not for a student.

What motivates some professors or students to create some problems and why some kinds of created problems are more appealing to some and not to others? So we begin to participate in a dialogue that enables people to reflect on what they value and how they think. Then many other questions (sometimes too philosophical) are considered, such as: Am I interesting in pursuing this problem? Do I understand this problem? What would it take for me to understand it better? Why am I being presented with this problem at this time? Are there others who have a different interest in this problem than I? Why?

As an example of a good posed problem and a good problem solving we consider the problem of finding the sum of the natural numbers from 1 to 100. As the myths goes this is a problem that Gauss' teacher gave to keep the students occupied for an extended period of time. Of course, Gauss got the answer in less than a minute by noticing that the first and last numbers added to 101, the second and penultimate numbers added to 101, and so on. After this, it would not be difficult to generalize the solution to finding the sum of the first n natural numbers.

Even when a problem is presented together with a solution, the pair "problem-solution" could be a force to generate questions that have a humanistic focus beyond the solution itself. We could analyse a great deal of information about the time and place of its origin. It is also possible to investigate the problem in such a way that each of us can use whatever resources we have at our disposal and imagine what might have been the circumstances that gave rise to the idea, for example, with regard to Goldbach's famous conjecture, Fermat's last theorem or Euclid's proof of an infinite number of primes.

Group projects for the students can be centred on the symmetry, the Pythagoreans' work with figurate numbers [2]. We could use the computer algebra systems, for example, Maple to avoid routine calculus and to have more time for reflection. The system Maple includes facilities for interactive algebra, calculus, discrete mathematics, graphics, numerical computation and many other areas of mathematics [3].

Students quickly become intrigued with discovering patterns, tessellations, three-dimensional geometric interpretations of algebraic equations [1], obtaining sequences, and finding sums based on figurate numbers. We could encourage students to act like mathematicians and demonstrate a mathematician's need to invent proofs as a result of creative impulses and intellectual challenge. Students learn how proof enables mathematicians to comprehend the structure of their discipline, reveals connections between topics, and empowers mathematicians to unify mathematical theories that lead to new discoveries.

SUPREME BEAUTY IN MATHEMATICS

The interesting case for mathematics as an art is the possibility of regarding at least some of its products as objects of aesthetic enjoyment.

Mathematics is a language, using carefully defined symbols and notions, a science and an art, characterized by order and internal consistency, harmony and beauty. And the professors working together to improve mathematical education must explore the connections between mathematics and art, in particular, the idea of symmetry, in order to enlarge and enliven courses ranging from elementary mathematics to abstract mathematics. Mathematics should include experiences that help students to shift their thinking about mathematics and define mathematics as a study of patterns and relationships, a science and an art [4].

As an example, what are tessellations? It is a periodic drawing division, a rhythmic theme on a plane, an arrangement of regular or irregular polygons or some repeating figures that completely covers the plane without overlapping or leaving gaps. Why are we interested in tessellations? They look nice! They teach us mathematics!

The mathematics invites many explanations of concrete works of art, and the beautiful patterns in Islamic art inspire discussions on geometry and symmetry. In relation to the works of the Dutch artist Escher, it is possible to enter into conversations about mathematics at a relatively advanced level, such as the Poincaré disc model of the hyperbolic plane.

A concept of elegance in mathematics might include the following aspects: an elegant solution is relatively easy to understand; an elegant solution is brief; the solution involves a creation that is unexpected. Where might we look for elegance in mathematics? Usually elegance in mathematics associated with proofs and statements of theorems or conjectures. As a rule, the elegant proof or statement connects up concepts that at first glance could not look more unrelated.

MATHEMATICS APPLICATION

We live in a world that has been decisively shaped by the applications of mathematics. Mathematics will advance in response to practical challenges and its internal momentum of mathematical curiosity. New connections are discovered between apparently different fields: mathematics and arts, mathematics and architecture, mathematics and physics, mathematics and biology.

When the opportunity arises, it can be fruitful to incorporate examples from the arts, architecture or nature in the teaching of mathematics. For example, the arts with awareness use such mathematical notions as golden section, perspective and tessellations. We can apply the mathematical knowledge (golden section, Fibonacci numbers and tessellation) in architecture to create the beautiful buildings. Some wonderful objects in nature (nautilus shells, petals on flowers, pine cones, leaf arrangements) have a relation with the Fibonacci numbers.

In the teaching of mathematics it is important to get the abstract structures in mathematics linked to concrete manifestations of mathematical relations in the outside world.

The present world goes through the process of globalization. Nowadays practical application of mathematics often follows immediately after scientific discoveries. All these new developments make necessary a renewed thinking on ethical criteria for mathematical activities and teaching mathematics. This is moreover encouraged by the increasing disciplinary specialization. Without a doubt, mathematics education could ethically apply existing versions of mathematics. In addition, there is a need to have a much more interactive process of communication between scientists (not only mathematicians), professors and students on connections between mathematical discoveries and their ethical application.

References

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