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THE GUIDED REINVENTION I N THE TEACHING OF LINEAR ALGEBRA

НАПРАВЛЯЕМОЕ ПЕРЕОСМЫСЛЕНИЕ В ПРЕПОДАВАНИИ ЛИНЕЙНОЙ АЛГЕБРЫ

The paper describes how “the guided reinvention” (a technique introduced by G. Freudenthal and close to the genetic approach) can be applied in the teaching of linear algebra. The principles of constructing the course, the sequence of the study, as well as the design of the process of the teaching of linear dependence are considered.

В статье описывается, как «направленное переосмысление» (прием, введенный Г. Фрейденталем и близкий к генетическому подходу) может быть применено в преподавании линейной алгебры. Рассматриваются принципы построения курса, последовательность изучения, а также проектирование процесса обучения линейной зависимости.

Keywords: Guided reinvention, linear algebra, linear dependence, higher mathematics.

Ключевые слова: Направляемое переосмысление, линейная алгебра, линейная зависимость, высшая математика.

Introduction. Earlier (Safuanov, 2019a), we described how "guided reinvention" can be used in teaching some areas of abstract algebra, for example, for teaching the concepts of a normal subgroup and a quotient group, as well as an ideal and a quotient ring. This paper is about the use of "the guided reinvention" in the teaching of some topics of linear algebra. The term "the guided reinvention" goes back to H. Freudenthal (1972)). Recently, papers have appeared devoted to "the guided reinvention" in the teaching of linear algebra (see, e. g., Plaxco et al., 2018). However, in our opinion, these articles use a version of "the guided reinvention", which still has significant differences from the genetic approach. If the genetic approach requires that the learning should follow the natural paths of the origin and development of knowledge (and not necessarily the actual historical path), "the guided reinvention" in the works of researchers that follow the theoretical direction of RME ("Realistic Mathematics Education" is reduced to the use of "emergent models" (Grawemeyer, 1999), where the process of mastering the material is somewhat artificially modelled.

The purpose of this work is to show how to use "the guided reinvention" in a linear algebra course (for pedagogical institutes and universities) so that the process of mastering knowledge follows the most natural paths of their origin (without reducing them to the historical path). This is especially important in the education of future mathematics teachers because it can improve their views on mathematics and its teaching (Kardanova et al., 2014; Safuanov, 2019b).

The sequence of studying the linear algebra course

The difficulty in planning a linear algebra course lies in the fact that it is advisable to study it at the initial stage of study in universities, earlier than other mathematical courses. However, for example, the concept of linear (vector) space is already quite complex and abstract and even more complicated than the concept of a 2

group and would require a serious mathematical proficiency to fully master it. The same applies to the concept of linear mappings (transformations, operators). The concepts of linear dependence (independence) and the related concepts of linear hull, basis, and dimension are very abstract and difficult to fully master.

For any arrangement of the linear algebra course or sections of the higher algebra course devoted to linear algebra, we consider it mandatory to preliminary study the following elements:

1) Basic concepts and notations of mathematical logic and set theory (including information about mappings);

2) Method of mathematical induction, inductive definitions, abbreviated summation notation;

2) The basic information about algebraic operations, groups, rings and fields.

Next, it is appropriate to introduce matrices, vectors (as special types of matrices – row vectors and column vectors), linear operations on them, and immediately apply the matrices to solve systems of linear equations using the Jordan – Gauss elimination method (when the augmented matrix of the system is transformed to the reduced echelon form – each column that contains a nonzero entry usually made to be 1 has zeros not just below that entry but also above that entry). This can also be used to infer how the inverse matrix is calculated (when possible). Then a theory for solving homogeneous systems of linear equations in terms of vectors is constructed, and the concept of a fundamental system of solutions is introduced. Only after this we introduce determinants, motivating their introduction in the usual way – by first solving a system of two linear equations with two unknowns, then solving a system of three equations with three unknowns, and by induction generalizing the method of constructing determinants by expanding along the first line (thus, at this point stage, one can construct determinants without introducing permutations and their properties). After this, still without introducing the concept of linear dependence, we can introduce the concept of the rank of a matrix as the number of non-zero rows in its echelon form, as well as a method for calculating it using the method of minors. This is how we arrive to the Kronecker-Capelli theorem. After that, one can introduce the eigenvectors and eigenvalues of the matrix (without yet introducing the concept of linear transformation), show a way to find all eigenvectors related to one eigenvalue using the fundamental system of solutions of the corresponding system of linear equations.

We have described here the introductory part of the linear algebra course, which in any case can be studied in the first year or even in the first semester, without using the concept of linear dependence, or linear spaces and linear mappings, or permutations and their properties. 3

However, more complex concepts of the further course, in particular, the concept of linear dependence, for full and deep mastering require a guided reinvention using carefully prepared actions based on the genetic approach.

Earlier (Safuanov, 2005)], we suggested to build a methodological development of a system for studying the subject (a section of a course, an important concept or a system of concepts by two parts: 1) a preliminary analysis of the sequence, tools and teaching methods and 2) a specific design of the process of teaching.

The preliminary analysis consists of two stages: 1) genetic development of the subject and 2) analysis of the arrangement of the material and the possibilities of using various means of presenting the material and impact on students.

Genetic development of the subject, in turn, is the analysis of the subject matter from four points of view:

a) historical; b) logical; c) psychological; d) socio-cultural.

After two stages of analysis, it is necessary to design the process of studying subject matter. In this case, it is advisable to divide the study process into four stages:

1) Construction of a problem situation. 2) Statement of new naturally arising questions. 3) Conceptual-structural analysis and logical organization of educational material. 4) Development of applications and algorithms.

We consider here only the second part – designing the process of studying educational material using the example of studying a linear dependence.

To accomplish this task, we will first indicate the most, in our opinion, appropriate sequence for studying the subject matter, and after that the methods of teaching certain sections of the course.

The sequence of studying the linear algebra course

1) Construction of a problem situation.

Since the subspace is completely determined by the system of generators, it is advisable to minimize this system of generators. The question arises: in what case can the number of generators be reduced? This could be done if one of the vectors can be removed from the system of generators so that the remaining vectors again form a system of generators. This, in turn, occurs when one of the vectors can be expressed as a linear combination of the others. In this case that vector can be removed, and it is in this case that we call the system of vectors linearly dependent. Here is the "moment of truth" when the concept of linear dependence is born.

2) Raising new naturally arising questions.

Examples of such questions:

In what case can not a single vector be removed from the system of generators of a subspace? (This question leads to the concept of linear independence.) In what case are two vectors linearly dependent? In what case are they linearly independent? 4

In what case is a system consisting of one vector linearly dependent? In what case is it linearly independent? If a system of vectors is linearly dependent, can each vector be expressed as a linear combination of others? Can a linearly independent system have a linearly dependent subsystem? Can a linearly independent system contain a zero vector? and so on.

3) Conceptual-structural analysis and logical organization.

After the construction of the problem situation and the onset of the "moment of truth", the definition of linear dependence and independence can be strictly formulated, and carefully selected and naturally formulated properties of linearly dependent and independent systems can be proved.

4) Development of applications and algorithms.

It is advisable to solve a sufficient number of exercises on finding the basis and rank of a system of vectors (this will also be an exercise on equivalent transformations of systems of linear equations).

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ВОЗМОЖНОСТИ ЦИФРОВОЙ ПАНЕЛИ EDFLAT ДЛЯ ОСУЩЕСТВЛЕНИЯ ПЕРСОНАЛИЗИРОВАННОЙ ПОМОЩИ УЧАЩИМСЯ В ПРОЦЕССЕ АДАПТИВНОГО ОБУЧЕНИЯ

МАТЕМАТИКЕ THE CAPABILITIES OF THE EDFLAT DIGITAL PANEL TO PROVIDE PERSONALIZED ASSISTANCE TO STUDENTS IN THE PROCESS OF ADAPTIVE MATHEMATICS LEARNING

В статье раскрываются возможности встроенных инструментов цифровой панели EDFLAT в создании адаптивных учебных материалов по математике с учетом типов восприятия учащимися учебной информации. Отмечается роль таких учебных материалов в оказании персонализированной помощи учащимся как непосредственной, так и опосредованной. Приводятся примеры учебных адаптированных материалов по математике, созданные инструментами цифровой панели, учитывающие стили восприятия учащихся.

The article reveals the capabilities of the built-in tools of the EDFLAT digital panel in creating adaptive teaching materials in mathematics, taking into account the types of perception of educational information by students. The role of such educational materials in providing personalized assistance to students, both direct and indirect, is noted. Examples of educational adapted materials in mathematics, created by the tools of the digital panel, taking into account the styles of perception of students, are given.

Ключевые слова: адаптивное обучение, персонализированная помощь, цифровая панель, адаптивные учебные материалы, обучение математике.