

The Contribution of Piotr Matus to Computational Mathematics

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Piotr Matus was born on July 24, 1953 in the small town of Ross, which lies in the Western part of Belarus. On the occasion of his 60th birthday, this article recalls his contributions to the development of the numerical analysis and mathematical simulation in Belarus and outlines some of his scientific achievements.

He graduated from the Belarusian State University, Department of Applied Mathematics, in 1975 and started his scientific career in the Institute of Mathematics of the National Academy of Sciences where he currently holds a position of a deputy director. Here Matus earned his PhD degree in 1980, a few years later he became the Head of the Department of Numerical Simulation. At this time he joined the group of academician Alexander Samarskii,

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founder of the Soviet School of Computational Mathematics and Mathematical Simulation, who largely influenced Matus' becoming a famous specialist in numerical analysis. The result of his communication with Samarskii's group and collaboration with various outstanding Russian numerical analysts was his doctoral dissertation defended in 1995 in the Institute of Mathematical Simulation of the Russian Academy of Sciences and appearance of the Russian version of the book "Difference Schemes with Operator Factors" in co-authorship with Alexander Samarskii and Petr Vabishchevich in 1998. In 1996 he was promoted to a full professor of mathematics at the National Academy of Sciences of Belarus.

Matus successfully combines his scientific research with educational work at the Belarusian State University and the John Paul II Catholic University of Lublin, Poland, where he holds professor positions. He devotes much energy and attention to teaching and undertakes a lot of effort for searching talented young people and engaging them in science. Professor Matus directed and advised 18 PhD students and numerous master students both in Belarus and Poland.

He has given outstanding service to the scientific community in Belarus in the promotion of computational mathematics. He organized a regular seminar on computational and applied mathematics in Minsk. This initiative undoubtedly helped to strengthen and enhance the School of Computational Mathematics in Belarus. He has worked actively towards building bridges between Eastern and Western scientific schools in numerical analysis. Throughout his academic career, Matus maintained contacts with colleagues in many parts of the world. An important milestone in his scientific activity was the creation of the journal "Computational Methods in Applied Mathematics" (CMAM) in 2001. This became possible due to his close collaboration with Alexander Samarskii and support of many of his colleagues, famous numerical analysts from all over the world, especially Vidar Thomee. At present the journal CMAM has established itself as an authoritative scientific publication known in the numerical analysis community. For many years Matus has acted as a member of the editorial boards of seven international scientific journals in computational mathematics and computer science.

His passionate desire to create a serious scientific journal and join efforts of both Eastern and Western scientists in the field of numerical analysis led him to organize regular international conferences under the aegis of the journal CMAM. There were five such conferences in Belarus (2003, 2007), Lithuania (2005), Poland (2010), and Germany (2012). All of these conferences attracted many famous numerical analysts from all over the world. The sixth CMAM conference will be held in Austria, September 28–October 4, 2014.

Matus contributed to numerical analysis over 250 scientific publications including two monographs. In early stage of his scientific career Matus investigated difference schemes for boundary problems for nonlinear equations with partial derivatives [1,2,6,18]. In these works, he obtained interesting results on convergence of approximate solution to exact solution under unbounded nonlinearity.

Later he investigated adaptive in time grids [7–10]. In many cases, when numerically solving initial-boundary value problems for time-dependent equations, it may occur that in some spatial regions there is a need for a smaller time step than in other regions. By using different time steps inside the computing domains, it is possible to reduce considerably the amount of computing required for the mathematical modeling of problems with singularities. On the basis of the fractional step procedures Matus suggested a general method for solving both linear and non-linear time-dependent problems. He investigated the convergence and stability of the proposed algorithms.

Along with an adaptive discretization in time, a series of Matus' papers are devoted to higher-order finite-difference schemes on nonuniform grids in space. In particular, the paper [24] is devoted to the construction of adaptive grids for solving nonstationary problems of mathematical physics by the method of dynamic locally refining grids. In [21–23, 26] essentially new algorithms with higher orders of accuracy on nonuniform spatial grids are suggested. These algorithms use a three-point stencil and higher order of local accuracy is obtained by consideration of the truncation error in a non-calculated point. This idea is also used for the construction of finite-difference schemes for problems in arbitrary domains [27].

The algorithms described above for time-dependent problems use variable weights and can be reduced to the operator-difference algorithms with operator factors. For investigation and substantiation of new computational algorithms, it allows using a common theory of stability and correctness of operator-difference schemes developed by Samarskii and his successors to whom Matus belongs.

Matus obtained important scientific results in the stability theory of differential-operator equations and operator-difference schemes. The paper [25] deals with two- and three-layer difference schemes with variable weights which are used for solving the Cauchy problem for evolution equations

$$\frac{d^m u}{dt^m} + Au = \varphi, \quad m = 1, 2$$

of the first and second order, respectively. There were found sufficient conditions for the stability of the difference schemes.

In the investigation of accuracy for evolutionary problems, Matus gave special attention to the estimation of the difference solution in grid analogs of time-integral norms. In [19], *a priori* estimates in such norms have been obtained for two-level operator-difference schemes. The use of these estimates is illustrated by the convergence investigation for schemes with weights for parabolic equations with solutions belonging to $W_2^{2,1}(Q_T)$.

Since the coefficients of the time-dependent problems are the input data, along with stability with respect to initial data and right-hand side, the stability with respect to perturbation of operator coefficients (strong stability) should be studied. In Matus' works, for the first time strong stability for time-dependent problems was investigated. It became possible because of the use of estimates in integral with respect to time norms. The papers [3–5] are devoted to the investigation of such problems. Some results, concerning stability and well-posedness of the operator-difference schemes, were published in [20].

Another avenue of Matus' research is the exact difference schemes for time dependent problems. Exact finite-difference schemes for hyperbolic and nonlinear parabolic equations whose solutions have the form of a running wave with constant velocity, $u(x, t) = \Psi(x - at)$, were constructed in [14–16]

One of the directions of Matus' theoretical work is the investigation of well-posedness and blow up for IBVP for semilinear parabolic equations and numerical methods. In [11–13, 17] he has obtained simple sufficient input data conditions, in which the solutions of differential and difference problems are globally bounded for all $0 \leq t \leq +\infty$. It was shown that, if these conditions are not satisfied, then the solution can blow up (go to infinity) in finite time. A lower bound for the blow-up time has been determined.

It is difficult in such a brief article as this one to recount all his achievements in the field of numerical methods and mathematical simulation. From the bottom of our hearts we wish our friend, Piotr Matus, on his jubilee great creative initiatives, new ideas and the implementation of the new achievements.

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