

TEACHING AND LEARNING MATHEMATICS HUMANISTICALLY

O.A. Barkov
Belarusian State Pedagogical University (Minsk)

Usually, a professor of mathematics shows a few examples, and assigns students some similar exercises for a drill. As a result, many students come to a view that mathematics is useless in real life, boring, and incomprehensible. Teaching mathematics humanistically demands creativity of both professor and a student. This theme emphasizes the meaning of emotional climate for learning mathematics. There are lots of promising ideas: historical background showing mathematics as a human discipline, interesting problems and open-ended questions, supreme beauty in mathematics, working in groups and teaching by projects, using of a computer algebra systems, mathematics applications in business, economics, art, and architecture. Which way should we go?

Historical background

How can we design mathematical courses that encourage students for excitement about mathematics connections in mathematics? One approach is to introduce a discussion via topics familiar to students and to expand the dialogue with historical references and activities that lead to a discovery of new directions or themes.

An example of a surprising fact in mathematics can be found in connection with regular polyhedrons. Pythagoras in the 500s BC knew that there can exist only five types of regular polyhedrons with respectively 4 (tetrahedron), 6 (cube), 8 (octahedron), 12 (dodecahedron) and 20 (icosahedron) polygonal lateral faces. The regular polyhedrons were associated with the four elements in ancient philosophy: fire (tetrahedron), air (octahedron), earth (cube), water (icosahedron). The dodecahedron was associated with an image of the universe itself.

It is not difficult to speculate exist upon possibility of existence of more than five regular polyhedrons. No kinds of experiments are sufficient to get a final answer to this question. It can be settled by a mathematical proof. A proof can be based on the theorem that the alternating sum of faces, edges and polygonal faces in the surface of a convex polyhedron equals 2, stated by Euler.

PROBLEM SOLVING AND GROUP PROJECTS

First of all, a problem is a non-routine task which is being encountered by the students for the very first time and, therefore, there is no obvious algorithm for the student to use. There are two kinds of mathematical problems: problems to find the unknowns and problems to prove which conjecture is true.

After this, a problem is a task which has certain open questions that challenge the student emotionally and intellectually. A problem is relative to students involved, i.e. what is a problem for one student may be an exercise for another. For example, the task to solve the equation $x^2 - 1 = 0$ may be a problem for a schooler but not for a student.

What motivates professors or students to create problems and why some kinds of problems are more appealing to one than to others? So we begin to participate in a dialogue that encourages people to reflect on what they value and how they think. Then many other questions (some of them philosophical) are considered, such as: Am I interesting in pursuing this problem? Do I understand this problem? What would it take for me to understand it better? Why am I being presented with this problem at this time? Are there others who have a different interest in this problem than I? Why?

As an example of a good posed problem and a good problem solving we consider the problem of finding the sum of the natural numbers from 1 to 100. As the myths goes this is a problem that

In less than a minute by noticing that the first and the last numbers added to 101, the second penultimate numbers added to 101, and so on. After this, it would not be difficult to generalize the way for finding the sum of the first n natural numbers.

Even when a problem and a solution are both presented, the pair "problem-solution" would force to generate questions that have a humanistic focus for the solution itself. We could analyze a great deal of information about the time and place of its origin. It is also possible to investigate the problem such a way that each of us can use any resources we have at our disposal and imagine what might have been the circumstances that gave rise to the idea, for example, with regard to Gauss's sum conjecture, Fermat's last theorem or Euclid's proof of an infinite number of primes.

Group projects for students can be centred on the symmetry, the Pythagorean work on figurate numbers [2]. We could use computer algebra systems, for example, Maple to avoid raw calculus and to have more time for reflection. The system Maple includes facilities for interactive algebra, calculus, discrete mathematics, graphics, numerical computation and many other areas of mathematics [3].

Students quickly become intrigued with discovering patterns, tessellations, three dimensional geometric interpretations of algebraic equations [1], obtaining sequences, and finding sums based on figurate numbers. We could encourage students to act like mathematicians and demonstrate the mathematician's need to invent proofs as a result of creative impulses and intellectual challenge. Students learn how to prove enables mathematicians to comprehend the structure of their discipline. It reveals connections between topics, and empowers mathematicians to unify mathematical theories and lead to new discoveries.

SUPREME BEAUTY IN MATHEMATICS

An interesting case for mathematics as an art is a possibility of regarding at least some of its products as objects of aesthetic enjoyment.

Mathematics is a language, that uses carefully defined symbols and notions, a science and an art, characterized by order and internal consistency, harmony and beauty. Professors working together to improve mathematical education must explore connections between mathematics and art, in particular the idea of symmetry, in order to enlarge and enliven courses ranging from elementary mathematics to abstract mathematics. Mathematics should include experiences that help students to shift their thinking about mathematics and define mathematics as a study of patterns and relationships, a science and an art [4].

As an example, what are tessellations? It is a periodic drawing division, a rhythmic theme or theme, an arrangement of regular or irregular polygons or some repeating figures that completely cover the plane without overlapping or leaving gaps. Why are we interested in tessellations? They teach us mathematics!

The mathematics can explain works of art. Beautiful patterns in Islamic art inspire discussions on geometry and symmetry. In relation to the works of the Dutch artist Escher, it is possible to discuss mathematics at a relatively advanced level, such as the Poincaré disc model of the hyperbolic plane.

A concept of elegance in mathematics might include the following aspects: an elegant solution is relatively easy to understand; an elegant solution is brief; it involves an unexpected creation. Where do we look for elegance in mathematics? Usually elegance in mathematics associated with proofs or statements of theorems or conjectures. As a rule, the elegant proof or a statement connects up concepts that at a first glance could not look more unrelated.

Mathematics application

We live in a world that has been decisively shaped by the applications of mathematics. Mathematics will advance in response to practical challenges and its internal momentum of mathematical discovery. New connections are discovered between apparently different fields: mathematics and art, mathematics and architecture, mathematics and physics, mathematics and biology.

It can be fruitful to incorporate examples from the arts, architecture or nature in the teaching of mathematics. For example, such mathematical notions as golden section, perspective and tessellations are used in art. We can apply knowledge in mathematics (golden section, Fibonacci numbers or tessellation) to architecture for creation of beautiful buildings. Some wonderful objects of nature (fractal shells, petals on flowers, pine cones, leaf arrangements etc) are related to the Fibonacci numbers.

In teaching mathematics it is important to get the abstract structures linked to concrete manifestations of mathematical relations in outer world.

The present world goes through the process of globalization. Nowadays practical applications of mathematics often goes immediately after scientific discoveries. All these new developments require necessarily of a renewed thinking on ethical criteria for mathematical activities and teaching mathematics. It is moreover encouraged by the increasing disciplinary specialization. Without a doubt, mathematics education could ethically apply existing versions of mathematics. In addition, there is a need to work with more interactive processes of communication between scientists (not only mathematicians), professors and students on connections between mathematical discoveries and their ethical implications.

References

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