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Remarks on hyperHermitian manifolds

We consider an almost hyperHermitian structure (ahHs) on a manifold M consisting of (J_1, J_2, J_3, g), where $J_i^2 = -I$, $J_1 J_2 = -J_2 J_1 = J_3$, $g(J_i X, J_i Y) = g(X, Y)$, $i = 1, 2, 3$, $X, Y \in \chi(M)$. If ∇ is the Riemannian connection of the Riemannian metric g, then the canonical connection $\overline{\nabla}$ in the sense of [**1**] of the ahHs has the following form

$$\overline{\nabla}_X Y = \frac{1}{4}\left(\nabla_X Y - J_1 \nabla_X J_1 Y - J_2 \nabla_X J_2 Y - J_3 \nabla_X J_3 Y\right), \qquad X, Y \in \chi(M).$$

In particular, $\overline{\nabla} g = 0$, $\overline{\nabla} J_i = 0$, $i = 1, 2, 3$.

The tensor field $h = \nabla - \overline{\nabla}$ is called the second fundamental tensor field of ahHs, [**1**].

Using results in [**2**] we have got the following

Theorem 1. *Let (M, J, g) be an almost Hermitian manifold. Then there exists a neighborhood $N_\triangle$ of the diagonal $\triangle(M \times M)$ in $M \times M$ that the manifold $N_\triangle$ has an ahHs (an infinite dimensional set of ahHs).*

Theorem 2. *Let (M, g) be a Riemannian manifold, then there exists a neighborhood $N_\triangle$ of the diagonal $\triangle(M \times M \times M \times M)$ in $M \times M \times M \times M$ that the manifold $N_\triangle$ has an ahHs (an infinite dimensional set of ahHs).*

Theorem 3. *There exists such an ahHs (J_1, J_2, J_3, g) that $\nabla J_3 = 0$ and $h_X Y = \frac{1}{2} g(\xi, X) J_3 Y$, where $||\xi|| \neq 0$, $\xi, X, Y \in \chi(M)$. If there exists a solution α of the equation $\xi = grad\alpha$, then the structures (J_i', g), $i = 1, 2, 3$ are Kaehlerian, where the ahHs (J_1', J_2', J_3', g) on M is defined by the following equalities*

$$\begin{aligned} J_1' &= \cos\alpha J_1 - \sin\alpha J_2, \\ J_2' &= \sin\alpha J_1 + \cos\alpha J_2, \qquad \alpha \in F(M). \\ J_3' &= J_3, \end{aligned}$$

Theorem 4. *Let we have such an ahHs (J_1, J_2, J_3, g) that $\nabla J_3 = 0$ and $h_X Y = \frac{1}{2} g(\xi, X) J_3 Y$, $L = [\xi]$. The ahHs is quasi homogeneous one,* [**1**], *if and only if $\overline{\nabla}\xi = 0$ on M. If the ahHs is quasi homogeneous then the distribution $L^\perp$ is integrable and its maximal integral manifolds are totally geodesic submanifolds with respect to ∇.*

References

[**1**] A. A. Ermolitski, Riemannian manifolds with geometric structures (monograph), Minsk: BSPU, 1998, 195 pp. (in Russian)

[**2**] S. A. Bogdanovich, A. A. Ermolitski, On almost hyperHermitian structures on Riemannian manifolds and tangent bundles, Central European Journal of Mathematics (to appear)