ГЕОМЕТРИЯ И ТОПОЛОГИЯ

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ON AN ALMOST HYPERHERMITIAN STRUCTURE ON A LOCALLY K-SYMMETRIC RIEMANNIAN SPACE

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Definition [1]. *A connected Riemannian manifold (M, g) with a family of local isometries $\{s_x : x \in M\}$ is called a locally k-symmetric Riemannian space (k-s.l.R.s.) if the following axioms are fulfilled:*

a) $s_x(x) = x$ and x is the isolated fixed point of the local symmetry s_x;

b) the tensor field S: $S_x = (s_{x*s})$ is smooth and invariant under any local isometry s_x;

c) $S^k = I$ and k is the least of such positive integers.

If M is a k-s.l.R.s. and $X, Y \in \chi(M)$ then the unique canonical connection $\widetilde{\nabla}$ can be defined by the following formula (see [2])

$$\widetilde{\nabla}_X Y = \nabla_X Y - \frac{1}{k}\sum_{j=1}^{k-1} \nabla_X (S^j) S^{k-j} Y = \frac{1}{k}\sum_{j=0}^{k-1} S^j \nabla_X S^{k-j} Y. \quad (1)$$

Further, we have $\widetilde{\nabla} g = \widetilde{\nabla}\widetilde{R} = \widetilde{\nabla} h = \widetilde{\nabla} S = 0$, $S(\widetilde{R}) = \widetilde{R}$, $S(h) = h$, $S(g) = g$, where $h = \nabla - \widetilde{\nabla}$ and $\widetilde{R}$ is the curvature tensor field of $\widetilde{\nabla}$.

Let M be such a k-s.l.R.s. that $S_x = (s_x)_{*x}$ has only complex eigenvalues $a_1 \pm b_1 i, \ldots, a_r \pm b_r i$. We define distributions D_i, $i = 1, \ldots, r$ by

$$D_i = ker(S^2 - 2a_i S + I).$$

Every $X \in \chi(M)$ has the unique decomposition $X = X_1 + \ldots + X_r$, where $X_i \in D_i$, $i = 1, \ldots, r$. An almost complex structure J on M is defined by

$$JX = \sum_{i=1}^{r} \frac{1}{b_i}(S - a_i I) X_i. \quad (2)$$

It is proved in [2] that (M, g) is an almost Hermitian structure and the connection $\widetilde{\nabla}$ defined by (1) coincides with the canonical connection $\overline{\nabla}$ of the pair (J, g).

Theorem. *Let (J_1, J_2, J_3, g) be such an almost hyperHermitian structure on a k-s.l.R.s. that $J_1 = J$ where J is defined by (2). In this case J_2 and J_3 are not invariant with respect to the family $\{s_x : x \in M\}$ and, therefore, with respect to the corresponding transitive pseudogroup of transformations of (M, g).*

References

[1] *Kowalski O.* Generalized symmetric space // Lecture Notes in Math. 805. – Springer-Verlag, 1980.

[2] *Ermolitski A.A.* Riemannian manifolds with geometric structures. – Minsk: BSPU, 1998.

DEFORMATIONS OF STRUCTURES ON A TUBULAR NEIGHBORHOOD OF A SUBMANIFOLD

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Let M' be a k-dimensional manifold isometrically embedded in a n-dimensional Riemannian manifold (M,g) and $Tb(M';\varepsilon)=\bigcup_{p\in M'} D(p;\varepsilon)$ be the normal tubular neighborhood of the submanifold M' in M. There exist coordinates $x_1,...,x_k$ in some neighborhood $V_0 \subset M$ of a point $o\in M'$ and for any point $x\in W_0=\bigcup_{p\in V_0} D_p$ there exists such unique point $p\in V_0$ that $x=exp_p(t\xi)$, $\|\xi\|=1$, $\xi\in T_pM'^{\perp}$. The point $x\in W_0$ has the coordinates $x_1,...,x_k,x_{k+1},...,x_n$ where $x_1,...,x_k$ are coordinates of the point p in V_0 and $x_{k+1},...,x_n$ are normal coordinates of x in D_p. We denote $X_i=\frac{\partial}{\partial x_i}$, $i=\overline{1,n}$, on W_0.

Let K be a smooth tensor field of type (r,s) on the manifold M and for $x\in W_0$

$$K_x=\sum_{i_1,...,i_r,j_1,...,j_s} k^{i_1,...,i_r}_{j_1,...,j_s}(x)X_{i_{1x}}\otimes...\otimes X_{i_{rx}}\otimes X_x^{j_1}\otimes...\otimes X_x^{j_s},$$

where $\{X_x^1,...,X_x^n\}$ is the dual basis of $T^*_x(M)$, $x=exp_p(t\xi)$, $\|\xi\|=1$, $\xi\in T_pM'^{\perp}$.

We consider a tensor field $\overline{K}$ on M

$$\overline{K}_x=\sum_{i_1,...,i_r,j_1,...,j_s} k^{i_1,...,i_r}_{j_1,...,j_s}(z)X_{i_{1x}}\otimes...\otimes X_{i_{rx}}\otimes X_x^{j_1}\otimes...\otimes X_x^{j_s}$$

by the following cases

a) $x\in D(p;\frac{\varepsilon}{2})$, $z=p$; b) $x\in D(p;\varepsilon)\setminus D(p;\frac{\varepsilon}{2})$, $z=exp_p(2t-\varepsilon)\xi$; c) $x\in M\setminus Tb(M';\varepsilon)$, $z=x$.

It is easy to see the independence of the tensor field $\overline{K}$ on a choice of coordinates in W_0 for every point $o\in M'$.

The tensor field $\overline{K}$ is called a *deformation of the tensor field* K *on a tubular neighborhood of the submanifold* M'.

The obtained tensor field $\overline{K}$ is continuous but is not smooth on the boundaries of $Tb(M';\frac{\varepsilon}{2})$ and $Tb(M';\varepsilon)$, $\overline{K}$ is smooth in other points of M.