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# MAXWELL EQUATIONS AND DIRECT CURRENT LAWS IN COMPLICATED NONSYMMETRIC ELECTRIC CIRCUITS

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Maxwell's equations, as basic ratio describing electromagnetic processes in conjunction electric and magnetic fields allow in some special case to analyze the appearance of a steady magnetic field in a conductive medium at the flow of direct current, but do not explain the magneto electricity phenomena of so-called ferroelectrics, in which the magnetization leads to an appearance of electrical polarization and vice versa.

$$\begin{aligned} \operatorname{rot} \vec{E} &= -\frac{\partial \vec{B}}{\partial t}, \\ \operatorname{rot} \vec{H} &= \vec{j} + \frac{\partial \vec{D}}{\partial t}, \\ \vec{B} &= \mu \mu_0 \vec{H}, \\ \vec{D} &= \varepsilon \varepsilon_0 \vec{E}, \\ \vec{j} &= \sigma \vec{E}. \end{aligned}$$

Here  $\vec{E}$ ,  $\vec{D}$  and  $\vec{H}$ ,  $\vec{B}$ , are the vectors of intensity, induction of electric and magnetic fields respectively,  $\varepsilon$  and  $\varepsilon_0$  are the dielectric permeability and electric constant,  $\mu$  and  $\mu_0$  are the magnetic permeability and the magnetic constant,  $\rho$  is the bulk density of free charges,  $\vec{j}$  and  $\sigma$  are the conduction-current density vector and electric conductivity of an isotropic medium. In ferroelectrics the stationary magnetoelectric phenomena could be formally reflected in the framework of the adopted system provided that there are additional equations for the relationship between the magnetization and the electric field vector in the medium, as well as between the electric polarization and the vector of the magnetic field intensity.

$$\vec{P}_i = \alpha_{ij} \vec{H}_j, \quad \vec{M}_i = \alpha_{ij} \vec{E}_j.$$

$\vec{P}_i$  and  $\vec{M}_i$  – are the polarization (the dipole moment per a unit volume) and the magnetization (the magnetic moment per a unit volume) of the medium,  $\alpha_{ij}$  – is a second-rank tensor of dimension  $[\text{C m}^{-1}]$ . The magnetoelectric effect, which can be longitudinal and transverse, corresponds to the diagonal and off-diagonal components of tensor  $\alpha_{ij}$ .

In the case of a stationary electromagnetic field the first two equations of the system are used in frame of so-called Kirchhoff rules relating to the operation of the electric field at charge transfer along a closed contour, and the law of conservation of charge. The sum of the

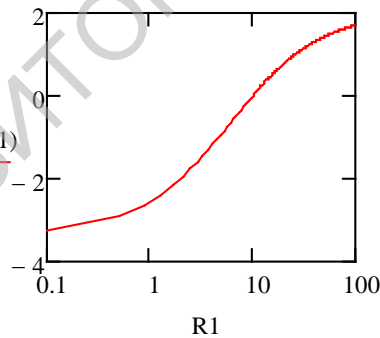
voltage drops along a closed contour is zero, the voltage drop at the source is equal to its electromotive force with the opposite sign, and the algebraic sum of the currents at any node of the circuit is zero. These rules make it convenient to use the methods of matrix algebra to calculate electrical circuits of a complex configuration, when the symmetry conditions can not be applied.

Here we represent the advantages of calculation of direct currents circuits in the formalism of Kirchhoff's laws allowing one to determine the characteristics of complex mixed parallel-sequential connections of elements un the example of one traditional scheme corresponding to the Winston bridge, which is widely used in metrology were a control zero current through the galvanometer in the diagonal is used. For this bridge having diagonal resistor  $R_3$  the initial system is

$$\begin{aligned} R_1 I_1 + 0I_2 + 0I_3 + 0I_4 + R_5 I_5 &= E, \\ 1I_1 + 0I_2 + 1I_3 + 0I_4 - 1I_5 &= 0, \\ 0I_1 + 1I_2 - 1I_3 - 1I_4 - 0I_5 &= 0, \\ R_1 I_1 - R_2 I_2 - R_3 I_3 + 0I_4 + 0I_5 &= 0, \\ 0I_1 + 0I_2 + R_3 I_3 - R_4 I_4 + R_5 I_5 &= 0. \end{aligned}$$

The current through a diagonal resistor  $R_3$  is the next

$$I_3 = \frac{E(R_1 R_4 - R_2 R_5)}{R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_2 R_5 + R_1 R_3 R_4 + R_1 R_4 R_5 + R_2 R_3 R_5 + R_2 R_4 R_5 + R_3 R_4 R_5}.$$



*Fig. 1.* The magnitude of the current through the diagonal of the Winston bridge in a function of controlling resistance  $R_1$  when all other resistors are of 10 Ohm and voltage supply is of 10 V.

So the formalism of Kirchhoff's laws allows to determine the characteristics of complex connections of elements by the formation of the initial system with the number of equations corresponding to the number of unknown parameters and subsequent correct mathematical operations.