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NOVEL CONCEPT OF CONCENTRATOR OF MAGNETIC ENERGY

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ABSTRACT

This paper presents a novel way of improving the magnetic system arrangement and efficiency via application of the Hall drift for the generation of additional magnetic field and an increase in the magnetic energy density. Such an increase may be realized with the use of a hyperconducting pure aluminum concentrator installed into a traditional solenoid. The Hall drift stimulated by the Lorentz force in crossed electric and magnetic fields has been investigated as a previously ignored source of magnetic field. The properties of the Hall drift of electrons and the generation of an associated self magnetic field are examined with aluminum conductors. An experiment on aluminum disk-like conductors with radial current between the inner and outer concentric contacts has been performed in an axial magnetic field. An analytical study has been made on the fundamental relations of charge flow in electric and magnetic fields under appropriate boundary conditions. The benefits and advantages of the application of the Hall drift magnetic field are discussed for the cryogenic temperature range.

INTRODUCTION

The problem associated with the design of effective cryogenic energetic devices and systems further extends the requirement for a geometrical optimization for containing most of the magnetic field in the same volume for a given energy supply. The high conductivity of pure metals at low temperatures ensures a strong outside layer of alternating current that is already at low frequency. The skin depth in such a conductor fabricated of pure aluminum with an *RRR* of 10000 (resistivity at liquid helium is 2×10^{-10} Ohm·cm) at a frequency of 1 Hz is only 0.1 cm. Thus, such metals as Cu, Al, Bi, and In may be utilized to increase the energy concentration of the nonstationary magnetic field by means of increasing the local magnetic field strength. Actually, a nonstationary magnetic field flux through a contour A with a cross section S_A may be transformed into the field flux through contour B with a smaller cross

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section S_B. The magnetic field magnitude H_b in contour B will be obtained from $H_B = S_A H_A / S_B$ where H_A is a magnetic field in contour A. A cylindrical hollow conductor having a radial split through out its length is a concentrator of the magnetic field energy. The phenomenon of such a concentration may be applied in cryogenic asynchronous electrical motors for exciting the alternating magnetic field in stator coils. The weakest part of such a concentrator is a split which decreases the mechanical hardness of the active element.

For stationary magnetic fields there is still another mode of magnetic field increase, namely the phenomenon of the Hall drift in a transversely unlimited conductor. In this paper we describe a novel concept of hyperconducting concentrator fabricated of high purity normal metal. This concept is based on the Hall drift of carriers stimulated by the Lorentz force in crossed electric and magnetic fields. It has been shown that the transverse Hall drift may occur in a cylindrical conductor placed in a coaxial external magnetic field when the current is made to flow between the inner and outer concentric leads.[1-3] In this case the cylindrical active conductive element with no split may be represented as a magnetic field amplifier or concentrator because additional movement of the carriers generates excessive magnetic field for the same energy supply.

EXPERIMENTAL AND ANALYTICAL PROCEDURE

The present investigation is both experimental and analytical. Aluminum was the material used in this study. The specimens used in the study were disks of different thickness (0.5 to 4 mm) and inner and outer diameters of 3 to 6 and 28 to 36 mm, respectively. The aluminum used had a polycrystal structure with an RRR of approximately 10000. The investigation were made in a regime of current supply where the maximum value of current density flowing through the sample between the inner and outer concentric contacts achieved a value of 1600 A/cm^2. An external magnetic field with a magnitude up to 85 kOe was directed normally to the end surface of the disk shape sample. Samples were placed in liquid helium with heat removal by vaporization of the liquid. A thermostat with automatic monitoring of the temperature was used for temperature measurements.[4-6]

Data acquisition involved determination of the integral magnetoconductivity, magnetoresistance, and self magnetic field in the disk center with an aid of Hall converters. Self magnetic field H_s was obtained as a difference between the total magnetic field H and the external field H_0 generated by the superconducting solenoid.

An analytical investigation has been made to provide basic relationships for current flow in a conductor having a cylindrical geometry. The main feature of this approach involves an equipotentiality of azimuthal contours due to the symmetry of the concentric inner and outer current contacts. This equipotentiality takes place both in the absence and in the presence of an external magnetic field for the polycrystalline material. To estimate a form of the current flow we used a charge discontinuity condition and a relation that connected the magnetic field with the current density as given by

$$\frac{1}{r}\frac{\partial}{\partial r}(r j_r) + \frac{\partial}{\partial \theta} j_\theta + \frac{\partial}{\partial z} j_z = 0; \quad \nabla \times \boldsymbol{H} = \frac{4\pi}{c}\boldsymbol{j}; \quad j_i = \sigma_{ik} E_k \qquad (1)$$

Here $\boldsymbol{j}$ is a current density vector, $\boldsymbol{H}$ a magnetic field strength vector, c a light velocity, j_i and E_k components of the current density and electric field in cylindrical coordinates, respectively. The conductivity and resistivity tensor components, σ_{ik} and ρ_{ik}, were also used in the analysis.

RESULTS AND DISCUSSION

Theoretical Study

Results of the analytical investigation show that the Hall drift of carriers in such a cylindrical conductor is determined by the type of Fermi surface encountered, i.e., by the degree of electron and hole concentration, and by the quantity of carrier on open directions of the Fermi surface. Open directions indicate those electron trajectories where a movement is infinite and almost straight.

By assuming the absence of current flow in the z direction, the current density of the Hall drift that is an azimuthal component j_θ is given by

$$j_\theta = -\frac{\rho_{\theta r} j_r}{\rho_{\theta\theta}} \tag{2}$$

For the compensated materials the conductivity tensor components are

$$\sigma_{rr} \sim \sigma_{\theta\theta} \sim \frac{1}{(\omega\tau)^2}; \quad \sigma_{r\theta} \sim \sigma_{\theta r} \sim \frac{1}{(\omega\tau)^2}; \text{ and } \sigma_{zz} \sim \sigma_0 \tag{3}$$

Here ω is a cyclotron frequency, τ the relaxation time, and σ_0 the conductivity in zero magnetic field. Under these conditions, the Hall drift current density is $j_\theta \approx j_r$.

For an open Fermi surface (for example, the open surface is along the r direction) $j_\theta \sim \omega\tau j_r$ because $\sigma_{rr} \sim (\omega\tau)^{-2}$, $\sigma_{\theta\theta} \sim const$, and $\sigma_{r\theta} \sim (\omega\tau)^{-1}$. But, if the direction of the open surface is along the θ direction, then $j_\theta \sim j_r(\omega\tau)^{-1}$. In a polycrystalline structural material with an open Fermi surface due to averaging the different electron directions, the transverse resistivity component is $\rho_{rr} \sim \rho_{\theta\theta} \sim \omega\tau$. Thus, the transverse current Hall density drift is of the same order of magnitude as the radial current density; that is, $j_\theta \sim j_r$.

For a closed Fermi surface, $\sigma_{rr} \sim \sigma_{\theta\theta} \sim (\omega\tau)^{-2}$ and $\sigma_{r\theta} \sim \sigma_{\theta r} \sim (\omega\tau)^{-1}$. Accordingly, $j_\theta \sim \omega\tau j_r$, which makes this type of electron structure most suitable for developing a concentrator of magnetic energy. For this reason a high purity aluminum with a polycrystalline structure was chosen for the experimental modeling of magnetic field concentration.

Approximation of Long Concentrator

The magnetic energy level depends on the total length of the cylindrical element.[7] Thus, an analytical study of energy concentration has been completed which involved two concentrator geometries, namely, the long concentrator and the short one. The long concentrator is a cylindrical element having a length that is much larger than the outer diameter. For such a conductor only the axial component of the magnetic field may be taken into account. Total magnetic field in the center of the concentrator is

$$H = H_0\left(\frac{r_2}{r_1}\right)^b, \quad b \approx \frac{2e\tau}{mc^2} I' \tag{4}$$

where H_0 is an external magnetic field, r_1 and r_2 the inner and outer radii, and I' a linear current density per unit of length.

This relation is only valid for the uncompensated material exhibiting a closed Fermi surface. Strong dependence of the magnetic field on the radius is a result of the magnetic nonlinearity during strong Hall drift when the azimuthal current density is much higher than the radial one. For metal with a $\tau = 10^{-10}$ sec, parameter $b \approx 10^{-3}I'/3$, when I' is given in A/cm.

Short Concentrator

The other limiting case of concentrator geometry, namely, the short disk-like conductor is characterized with its small length when compared with its diameter. In this geometry, the azimuthal and radial magnetic field components and their derivatives cannot be ignored. The calculation of the axial magnetic field component occurring in the sample center has to be completed with a superposition of the magnetic fields for each one of the current elements. Thus, the self magnetic field in the disk center is

$$H_z = H_0 \frac{2I'R}{c\rho_{00}} \left| -\ln\left(\frac{r_1 \left(\frac{h}{2} + \sqrt{r_2^2 + \left(\frac{h}{2}\right)^2} \right)}{r_2 \left(\frac{h}{2} + \sqrt{r_1^2 + \left(\frac{h}{2}\right)^2} \right)} \right) \right| \quad (5)$$

where $I' = I/h$, h is the thickness of the short cylindrical conductor, and R the Hall coefficient.

Experimental Data

Experimental work has been completed for the conductor with the geometry closed to the short variant. Figure 1 represents the magnetic field increase in the conductor center as function of the ratio of the total magnetic field H to the external magnetic field H_0. These data are shown as functions of the averaged radial current density j_r given as

$$j_r = \frac{I}{\pi h(r_1 + r_2)} \quad (6)$$

Since radial current flows by means of an external supply, the current density is an independent parameter. The total to external field ratio indicates the degree of magnetic field increase. It is interesting that the triangles corresponding to higher external magnetic fields have less magnitude at other similar conditions. The reason for this phenomenon is due to a behavior of the transverse resistivity in the magnetic field whereas the Hall coefficient is a fixed parameter. Also, a nonmonotonic behavior at high external magnetic field is observed. The source of such a dependence has to be related to the additional increase of the transverse resistivity.

Self magnetic field energy density as a function of external field is shown in Fig.2. Self field energy is an additional energy similar to that of the external field. It is shown that the character of energy density growth at a radial current density of 600 to 700 A/cm^2 is the most promising at an external field of 80 kOe as noted by the higher slope of a curve. For an external field of 40 kOe, the prospective radial current density is 1200 to 1400 A/cm^2. The saturation and nonmonotonic character of the curves is observed in Figures 1 and 2. This indicates that some mechanism exists which restricts the magnitude of the self magnetic field.

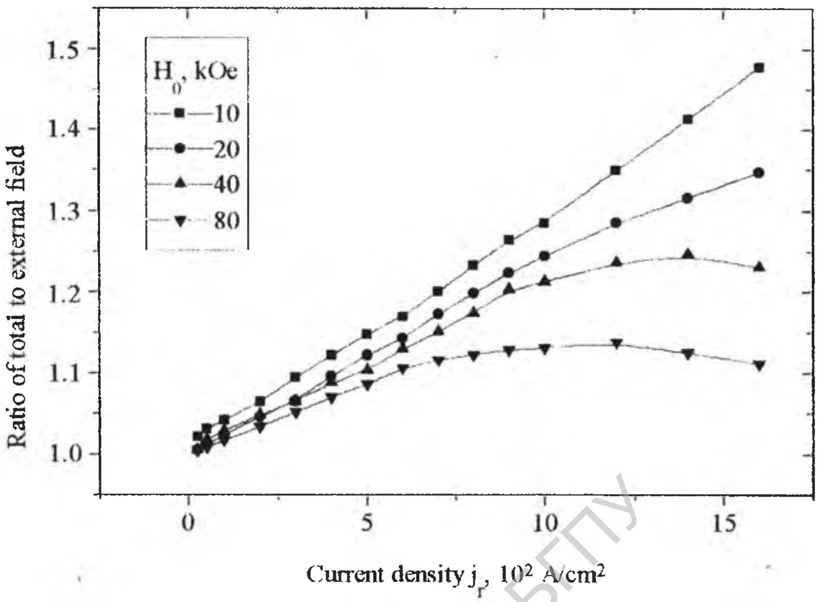

Figure 1. Ratio of the total magnetic field to an external one, H/H_0, versus the radial current density for the external field from 10 kOe up to 80 kOe.

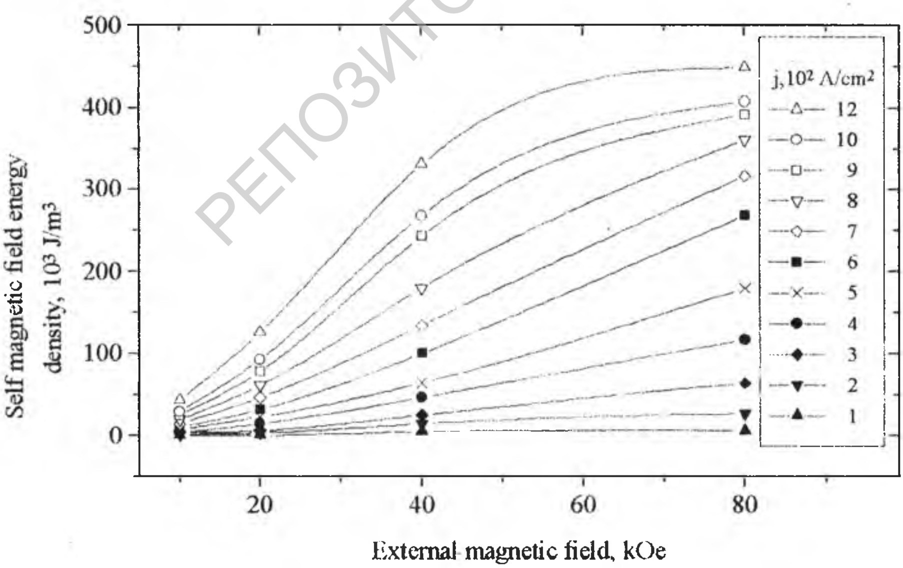

Figure 2. Self magnetic energy density in the disk center as a function of the external magnetic field for a radial current density from 100 to 1200 A/cm^2.

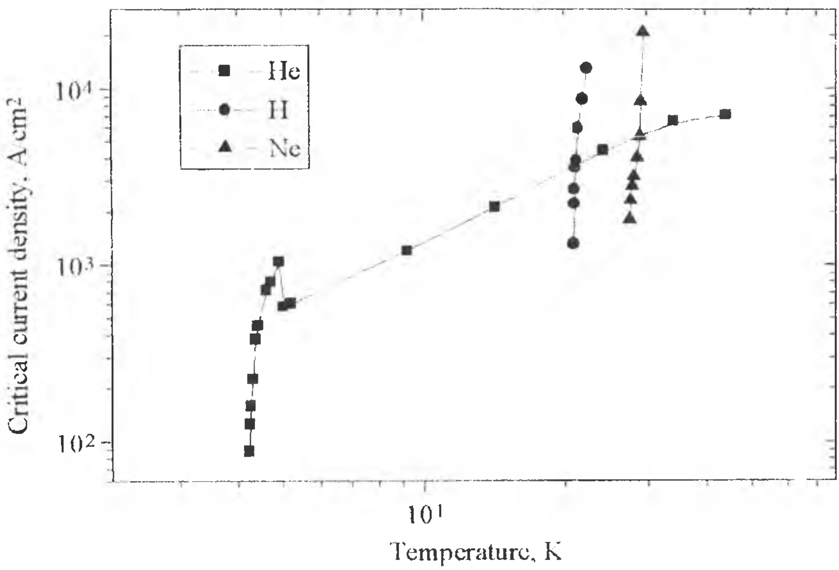

Figure 3. Critical current densities for different types of boiling for helium, hydrogen, neon.

Such limitation must be associated with the conductor thermal state, that is with dissipative processes in volume. In other words, an increase in conductor temperature occurs and as a result the azimuthal current density becomes smaller. The range of current density through the conductor is determined by the boiling curve of the cryogenic liquids. In this experiment we used liquid helium. The data show that at large current density the temperature of the conductor surface was higher than 4.2 K. Thus when the current density achieves a definite level, the helium can not provide a sufficient cooling and the temperature increases in accordance with the conditions of boiling. We have calculated a critical current density, based on our data, for three cryogenic liquids (helium, hydrogen and neon). Figure 3 shows the critical current density for the aluminum concentrator with the previously noted purity and structure versus the conditions of cooling. The data have been calculated based on the results obtained upon boiling He, H and Ne on the aluminum surface at different temperatures. The heat balance equation used is given by

$$\rho j^2 - \mathrm{div}\, q = 0 \qquad (7)$$

where q is the heat flux density vector through the conductor surface.

The critical current density denotes a maximum current density that may be supplied to the concentrator without a further temperature increase. The temperature dependence of the resistivity associated with a magnetic field magnitude of 80 kOe was taken into account in the calculation.[1]

CONCLUSION

A steady magnetic field concentration may be realized at cryogenic temperatures based on the strong electron Hall drift occurring in crossed electric and magnetic fields in hollow cylindrical conductors with an axial external magnetic field in which radial current is forced to flow between the inner and outer cylindrical contacts.

The extent of the magnetic field concentration is determined by the geometry (the most favorable is the long cylinder approximation), dispersion law, crystal structure, temperature and purity of the concentrator material.

Dissipative processes in hyperconductors restrict the range of concentration; however, the experimentally observed level of magnetic field increases by 50% even for the more unprofitable short construction. An increase in magnetic energy density of about 5×10^5 J/m^3 has been obtained.

Large helical toroidal magnetic systems supplied with currents on the order of 10 kA/cm^2 are suitable for installation in the cavity of the long variant of the concentrator. As a result, the power law of magnetic field increase may be achieved under moderate heat conditions.

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