# Nonlinear High-Frequency Effects in Metals at Heating Skin Layer

V.R.Sobol<sup>1</sup> and N.V.Frantskevich<sup>2</sup>

<sup>1</sup>Institute of Solid State and Semiconductor Physics NANB, 17 P.Brovka St., Minsk, 220726, Belarus E-mail: sobol@ifttp.bas-net.by

<sup>2</sup>Belarussian National Technical University, 65 F.Skarina Av., Minsk, 220027, Belarus

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The influence of the wave self-magnetic field on a movement of carriers and high-frequency current is considered under conditions when a wave phase velocity in metal is smaller than the Fermi velocity. It is shown that the existence of electrical nonlinearity of magnetodynamic nature is possible. In other words the nonlinear corrections in material equations are caused by influence of a magnetic field on moving charge carriers. The character of influence of degree of diffuseness on impedance is considered. It is shown that as opposed to usual Reiter-Sondheimer approximation in a mode of nonlinear dynamic connection the high-frequency current at a surface of metal is different at mirror and diffuse reflection of charge carriers. Conditions at which the external magnetic field results to skin-electron heating are determined.

Key words: abnornal skinning, normal skinning, Fermi velocity, phase velocity, surface reflection

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### **1** Introduction

Electromagnetic waves are substantially reflected by a surface of metal. They penetrate into it on small depth when external stationary magnetic field is absent [1], [2]. Particularly in optical range of frequencies i.e. when frequency is less of frequency of plasmas fluctuations only one characteristic length of attenuation of a field takes place. For range where  $\tau' \gg \omega^{-1}$  the relaxation time  $\tau$  is determined from the ratio  $\tau = \tau'(1 + i\omega\tau')^{-1}$ , here  $\tau'$  is a relaxation time under steady current flow,  $\omega$  is an electromagnetic field frequency. Skin depth is equal to the ratio of light velosity to plasma frequency  $\delta \cong m^{0.5}(\mu_0 n)^{-0.5} e^{-1}$ , here  $\delta$  is a skin depth, m is an electron mass, n is an electron concentration,  $\mu_0$  is a magnetic constant (4 $\pi 10^{-7}$  F/m), e is an electron charge.

In conditions of abnormal skinning there are two scales of lengths of field attenuation.

They correspond to depth of a skin-layer  $\delta \cong$  $(\rho l)^{1/3} (\omega \mu_0)^{-2/3}$  and to a free length of of carriers ( $\rho$  is a specific resistance, l is a free electron length). The big high-frequency current on depth of a skin-layer is created with the particles moving in a phase with a wave. Particles having an extreme value of normal velocity to a surface can penetrate in metal on even bigger depth. In other words the carriers moving deep into volume with a velocity being much greater than that of the order  $\omega\delta$  carry away the information on a field of a skin-layer on distance about free length of electron l. As a result there are two scales of attenuation. Respectively a layer of metal of thickness dis actually transparent if the condition  $\delta \ll d \leq l$ takes place [3], [4].

In this report a short review and analysis are done on the influence of magnetic field on electron frequency properties in materials having a metal type of conductivity. The frequency range is taken from radio area up to optical range. Analysis includes an influence of an external stationary magnetic field in combination with own magnetic and electric field component of wave.

# 2 Analytical procedure and discussion of results

Consider the influence of an external stationary magnetic field and own field of a wave on movement of charged particles. Define a range of light intensity resulting to nonlinearity of impedance. Use the general known phenomenological approach and equations for movement of a particle under combine action of steady magnetic field and magnetic component of wave [4], [5].

$$\frac{d\boldsymbol{p}}{dt} = e\boldsymbol{E} + e\left[\boldsymbol{v} \times (\boldsymbol{B_0} + \boldsymbol{B})\right]$$
(1)

Here  $B_0$  is an external stationary magnetic field, B is a variable component of magnetic field on a surface being connected to a magnetic field of a wave. Wave field has components E and H, v is a Fermi velocity, p is a Fermi pulse.

Estimate the contribution of a magnetic field of wave H(r,t) in Eq. (1) in comparison with electric that taking into account the condition  $B = E/v_p$  (here  $v_p$  is a phase velocity of wave in metal). It means that  $v_p$  corresponds with Fermi velocity in certain ratio. This ratio is defined by a character of connection between a current and a field. For a case of normal skinning it is possible to use approach [6], [7].

$$\frac{v_p}{v} = \frac{1}{v} \left(\frac{\omega\rho}{\mu_0}\right)^{1/2} \tag{2}$$

For the abnormal skin-effect

$$\frac{v_p}{v} = \frac{1}{v} \left(\frac{\omega^2 \rho l}{\mu_0}\right)^{1/3} \tag{3}$$

In normal metals such as Cu, Ag, Au, Al, etc. a velocity  $v \cong 10^6$  m/s. Respectively a relation  $v_p/v$  for an optical range of frequencies (frequency is about  $10^{15}$ ) is defined by size of specific resistance  $\rho$ . For low temperatures at high purity of material ( $\rho \cong 10^{-12}$  Ohm·m) the relation  $v_p/v$ has size about  $10^{-2}$ , and at room temperature it is higher on two order of value. It is typical for approximation of local connection in accordance with Eq. (2).

An approach of abnormal skin-effect for definition  $v_p/v$  in optical area is hardly justified (relation  $v_p/v$  following Eq. (2) is about 10). For radio-range ( $\omega \cong 10^6$ ) a relation  $v_p/v$  via (2) makes about  $10^{-6}$ . It is more realistic to use (3) which gives  $v_p/v \cong 10^{-5}$ .

Represent a movement of a particle in metal semi-space  $x \ge 0$  under an action of an external field  $B_0$  and also of E and H components of monochromatic wave as in [4], [8]. In approximation of plane waves an equation (1) corresponds to an expression

$$\frac{d\boldsymbol{p}}{dt} = e\left[\boldsymbol{v} \times \boldsymbol{B}_{0}\right] + e\boldsymbol{E} + \frac{e}{i\omega}\left[\boldsymbol{v} \times rot\boldsymbol{E}\right] \qquad (4)$$

Consider the first term in the right part of (4). Simple substitution shows that product  $evB_0$  for external field  $B_0 \cong 10$  T is about  $5 \cdot 10^{-12}$  n. For optical frequency range the size  $\omega p \cong 10^{-9}$ . In other words the influence on particle pulse of any experimentally achieved stationary external field  $B_0$  is very weak for an optical range of frequencies in comparison with its change under action of a wave field. At that time for a radio-range the product  $\omega p \cong 10^{-18}$  n and the reversal picture is observed. That is the change of a pulse under an action of a stationary field is more than influence of an electric field of a wave. From this estimations having been carried out it follows that in radio-area the external magnetic field will suppress the phenomenon of electric dynamic nonlinearity while in an optical range of frequencies its influence can be neglected.

Further consider own magneto-dynamic nonlinearity (for radio-area an external field is to be taken as equal to zero, for optical area the external field can be presented because it is negligible). Take into account that  $[v \times rot E] =$ 

Nonlinear Phenomena in Complex Systems Vol. 1, No. 1, 2004

 $-(\boldsymbol{v}\cdot\nabla)\boldsymbol{E}+\nabla(\boldsymbol{v}\cdot\boldsymbol{E})$ . Using this ratio it is possible to convince that expressions for the components of a particle pulse are fair.

$$p_{\alpha} + \frac{eE_{\alpha}}{i\omega} = Const; \quad \frac{dp_{x}}{dt} = \frac{ev_{\alpha}}{i\omega} \frac{\partial E_{\alpha}}{\partial x} \quad (5)$$

Here  $\alpha = a$  function of (y, z). Estimations show that for  $\omega \cong 10^{15} \text{ c}^{-1}$  the magnitude  $eE/\omega$  is compared on the order of Fermi pulse  $(10^{-24} \text{ kg} \cdot \text{m/s})$ at  $E \simeq 10^{10}$  V/m. This corresponds to giant intensity about  $10^{17}$  W/m<sup>2</sup>. At all values of intensity being smaller of mentioned above the movement of electrons in a plane of wave front does not differ from free movement. However the big value of Fermi velocity in comparison with phase velocity results that the derivative of a pulse of a particle along a normal to a surface is great. In other words there is a periodic movement of a charge along a normal to a surface. A periodic movement takes place in a field of a wave. An electric component of wave on a surface is about  $10^8$  V/m. Electron comes back with a magnetic field of a wave to a surface. Respectively time of its stay in a skin-layer grows. It is possible to assert that linear approach on electric field at low temperatures takes place when intensity of wave  $\leq 10^{12} \div 10^{13} \text{ W/m}^2.$ 

On the other hand, from a ratio (5) it follows, that for  $\omega \cong 10^6$  c<sup>-1</sup> the size  $eE/\omega$  is compared under the order of value with Fermi pulse at  $E \cong 10$  V/m. Therefore at intensity of an electric field of a wave less than 0.1 V/cm a movement of a charge in a plane of wave front a little differs from free movement. So a periodic movement in a direction of a normal begins for a wave with intensity on five orders smaller namely at intensity of the order of about  $10^{-11}$  W/m<sup>2</sup>. Under an action of a magnetic field of a wave an electron comes back to a surface of metal and time of its stay in skin-layer grows.

Thus for radio-area of frequency near  $10^6$  the realization of a nonlinear mode of operation demands to exclude an external magnetic field with accuracy up to about  $10^{-5}$  T. Otherwise it will lead to linearity under strong external magnetic

influence. In this range it is impossible to receive only electric nonlinearity and warm up a skin-layer. An electric nonlinearity at intensity of electric component E of about 10 V/m will be suppressed before with magnetic nonlinearity which begins at an intensity  $10^{-4}$  V/m. As it was mentioned above an external magnetic field is to be excluded.

As it is known in linear approximation at small amplitudes of an electric field of a wave the impedance for mirror and diffuse reflections of carriers by a surface differs only with a numerical multiplier about unit. In a nonlinear case the frequency current shunting an electromagnetic wave has a bigger size at mirror reflection than at diffuse reflection. Namely current at mirror reflection is larger of the current at diffuse reflection in a factor  $l/(r\delta)^{0.5}$  (here r is a Larmor radius). The dispersive equation in approximation of non-local connection corresponding simple linear approach is

$$k^2 \cong \frac{\omega \mu_0 \delta}{\rho l} \tag{6}$$

Under an influence of an external magnetic field

$$k^{2} \cong \frac{\omega\mu_{0}}{\rho l} \frac{\delta}{\varsigma + (r\delta)^{0.5} l^{-1}}$$
(7)

Here  $\varsigma$  is a parameter of diffusion. It takes meanings from zero up to unit. Accordingly at diffuse reflection a character of skinning poorly depends on intensity of a wave. At reflection being close to mirror that the high-frequency conductivity in addition grows by some orders of magnitude because of a curvature of trajectories in a wave field.

Warming up of electrons have not leaving a skin-layer becomes essential if energy which is received by particle on a free length becomes of order of characteristic value  $\varepsilon_F v_{ph}/v$  ( $\varepsilon_F$  is a Fermi energy,  $v_{ph}$  is a phonon velocity). In view of features of interaction with a surface it is possible to present this relation via characteristic parameters of carriers movement

Nonlinear Phenomena in Complex Systems Vol. 1, No. 1, 2004

$$\frac{eE(r_0\delta)^{0.5}}{\varsigma + (r_0\delta)^{0.5}l^{-1}} \ge v_{ph}p \tag{8}$$

Here  $r_0$  is a Larmor radius in external magnetic field  $B_0$ . At condition when an influence of  $B_0$ is so significant that  $\omega p \delta \gg eEr_0$  the magnetic field of a wave can be not taken into account and the next relation represents the frequency range where electron heating is possible

$$\omega \gg v_{ph} \frac{\varsigma + (r_0 \delta)^{0.5} l^{-1}}{\delta^{1.5}} r_0^{0.5}$$
(9)

At mirror reflection of electron from metal surface the condition (9) is reduced to an expression for frequency range of view  $\omega \gg \frac{v_{ph}r_0}{v\delta\tau}$ . For diffuse reflections the condition of heating of a skinlayer looks as  $\omega \gg \frac{v_{ph}r_0^{0.5}}{\delta^{1.5}}$ . For example for a pure metal at  $B_0 \cong 10$  T the respective value of relaxation time  $\tau \cong 10^{-10}$  s, the value of frequency of electromagnetic wave resulting to non-linearity for the account of heating corresponds  $10^9 \text{ s}^{-1}$  at mirror reflection and  $10^11 \text{ s}^{-1}$  at diffuse reflection of carriers from a surface.

## **3** Concluding remarks

The analysis of frequency phenomena of charge transfer in media having metallic type of conductivity shows that both an external stationary magnetic field and also the electric and magnetic components of electromagnetic field are able to stimulate electric non-linearity being transparent through dependence of surface impedance on parameters of task. The total range of observing of these properties spreads from radio area corresponding to a vacuum wave length of tens of meters up to optical frequency region corresponding to hundreds of nanometers. Respectively the range of realisation of non-linear properties are mainly determined with ratio between characteristic parameters having dimension of reversal time. That is three characteristic frequencies namely a frequency of electron scattering, a Larmor frequency and a frequency of electromagnetic field are to be compared. The largest of these parameters determines corresponding type of movement of particles and respective properties of material.

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Nonlinear Phenomena in Complex Systems Vol. 1, No. 1, 2004