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Hall current and losses in composite cryoconductor

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Abstract

We discuss the problem of steady field and current flow distribution in model double-strip composite conductor having the electric contact in its interface. Composite components are aluminum and anti-alumium - hypothetic conductor having opposite on sign Hall coefficient whereas all other components of its resistivity tensor are the same as aluminum resistivity tensor. The interface is orthogonal to an external magnetic field. Some new approaches for this problem are developed. The analysis is based on the phenomenological macroscopic equations for steady current field including the renewal of the hypothesis of existence the transverse current in every strip. The transverse current density value is accepted to depend on the distance to the interface plane. The transverse and transport current densities are obtained and the expression for the effective transport resistivity of composite conductor is estimated via the averaging of current density through the conductor volume. It is seen that the effective resistivity is a characteristics of an intermediate case between two limit geometries of charge flow. One limit geometry is a current flow through a single conductor (the interface does not influence on to current density allocation and resistivity). Another limit case is similar to current flow via disk like Corbino conductor (the interface influences on to charge flow very highly, the effective resistivity is large). It is shown that for relevant geometric parameters of double strip conductor the intermediate case for current density distribution and effective resistivity is closer to Corbino geometry than to single conductor geometry.

1. Introduction

Enhancement of magneto-resistance the composite conductors being a stabilizers of superconducting cables takes place during operation. The application of aluminum and of copper together in joint composite conductor is more profit than separately. Aluminum has very small magneto-resistance in comparison with other conductors of normal metals. Its Fermi surface is closed that and dispersion law is very similar to that of free electron gas. Some peculiarities are also present and as a result the resistance shows weak non saturation in magnetic field. Especially this takes place for polycrystalline pure metal sample. However the mechanical properties of aluminum are not satisfied because its

hardness is very small even at cryogenic temperatures. The copper is more mechanically strong metal and it is more suitable from this point of view but the electrical properties of copper are not very good. The Fermi surface of copper is of open type and as a result the resistance in magnetic field for polycrystalline conductor has strong linear dependence on magnetic field. This behavior have been observed firstly by Kapitza and named after his name. On this reason the combination of these components is used for manufacture different pool cooled superconductors for helical devices and magnets where the superconducting filaments for example NbTi are composed with aluminum stabilizers and a copper casing. An effective resistance of composite conductor at zero magnetic field is consistent with predictions for simple parallel circuit model where high pure core is shunted with high resistance jacket. However the results of measurement of resistance under magnetic field presence are different from the predictions of parallel circuit model. The magneto-resistance exceeds that expected, and it has large linear increase with magnetic field more similar for copper behavior instead of anticipated almost saturation behavior of aluminum. It was proposed that this anomalous behavior was a result of the existence of additional dissipation current that increases energy losses and generates a higher effective resistance for composite under the same value of current of source. This additional transverse current of Hall nature and its influence on transport properties of composite conductor have been analyzing in the whole file of papers, see [1–10].
The purpose of the present report is to develop some novel approaches for study the Hall current and improve the modeling of initial reason that generates an excessive magneto-resistance of composite conductor. Corbino geometry for charge movement and respective strong increase of resistivity due to annihilation of electric Hall field in aluminum disk sample is applied for analysis of the composite resistance problem [11]. Actually both current arrangement pictures are rather similar because in any that there is a possibility for appearance of closed electric lines and particle movement along these lines. Here we analyze the most simple variant of composite conductor consisting of the same in geometry strips whereas the material of strip is similar on its resistance in magnetic field.

2. Approach

Some novel approaches for this problem are developed. As the polarity of the Hall effect in aluminum is opposite to that in copper and this circumstance is the main reason of transverse current generation the prime model of composite is chosen as a double-strip that. Moreover the composite is chosen as consisted of components that have an ideal electric contact via the interface being orthogonal to an external magnetic field. The external magnetic field is rather strong (Larmor radius is much less of charged particle free length) as a result the own magnetic field of current may be neglected in composite volume. For the simplicity of analysis and to verify the approach and results of analysis the model type of conductor is used as a conductor consisted of components having very similar type of conductivity and resistivity tensor. In other words one of component is aluminum conductor with hole type of conductivity (Al^+), its Hall coefficient is positive, but another component is a hypothetical quasi-aluminum conductor having electron type of conductivity (Al^-) its Hall coefficient is the same on magnitude and negative on sign. As a result the conductivity and resistivity tensors of these components are the same in modulo excluding of diagonal linear on magnetic field components being opposite on sign.

The model composite of aluminum (Al^+) -anti-aluminum (Al^-) combination is more easy for analysis as the definite symmetry of conductivity properties simplify the contribution in to magneto-stimulated transverse Hall-like current generation of both composite components. The application of the tensor relations between current density and electric field in this case allows to regard only one half of composite conductor. The basic suggestion is that there is closed current lines in composite. These lines are a result of Loretce force action on to moving charged particle. Respective tensor relations have to be used under suitable boundary conditions.

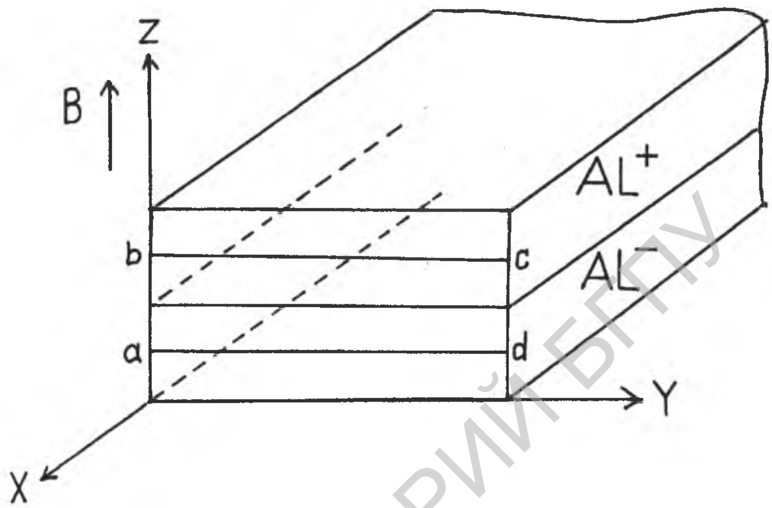

Fig.1 Composite double strip conductor and its arrangement in an external magnetic field.

Double strip composite conductor model is displayed in Fig.1. The conductor is uniform along transport X-direction and a primary electric field is applied along this direction. The magnetic field is oriented along Z-axis. The strips of conductor have the same dimensions. In steady current state the equations that govern the transport process and transverse Hall current mechanism generation in massive conductor (the cross dimensions are much higher of charge free length) are the averaged macroscopic phenomenological field relations of potentiality for steady electric field, current density continuity conditions closed with material conditions:

$$\nabla \times E = 0; \quad j = \sigma E; \quad E = \rho j \qquad (1)$$

here E is an electric field vector, j is a current density vector, σ is a conductivity tensor being equal to the reciprocal of resistivity tensor ρ. In this consideration:

$$\rho^{+}_{yx} = -\rho^{+}_{xy} = RB; \quad \rho^{-}_{yx} = -\rho^{-}_{xy} = -R$$

here ρ_{ik} is a component of resistivity tensor ρ, R is Hall coefficient, B – an external magnetic field.
We denote aluminum component as Al^+ and anti-aluminum component as Al^-.

$$\rho(A\uparrow) = \rho^{+}; \;\; \rho(A\downarrow) = \rho^{-};$$

$$\rho_{ik} = \begin{pmatrix} \rho_{xx} & \rho_{xy} & \rho_{xz} \\ \rho_{yx} & \rho_{yy} & \rho_{yz} \\ \rho_{zx} & \rho_{zy} & \rho_{zz} \end{pmatrix}; \qquad (2)$$

Here the largest terms of tensor components proportional to $(RB/\rho_{xx})^2$ are taken into account. It should be stressed that only two off-diagonal terms of resistivity tensor namely ρ_{xy} and ρ_{yx} are of great value under the magnetic field action, whereas all other components of ρ^+ and ρ^- are of the same order of magnitude of the resistivity at zero magnetic field. The components of conductivity tensor have more complicated dependence on magnetic field: transverse diagonal components are positive and are proportional to magnetic field to the power minus 2, whereas all off-diagonal components are proportional to inverse magnetic field.
The basic suggestion of this task in all papers have been numerated here before is that a primary transport electric field component E_x is a uniform that through the volume of double strip conductor. The total problem to be defined is the effective resistivity of composite conductor ρ_{ef} being the coefficient of proportionality between transport electric field component E_x and averaged through a cross section a transport current density $<j_x>$.

$$\rho_{eff} = \frac{E_x}{< j_x >} \qquad (3)$$

3. Calculation and results

On the base of tensor relations between current density and electric field components it is clear that the conditions for an existence of transverse current of Hall nature take place for the geometry of conductor. Really the usual electric Hall field is much higher of transport directional field for singular conductor. In the case of composite this high Hall field is shorten by conducting medium of another component. Respectively the electric Hall field of another component is shorten by the conducting medium of the first component so that Hall fields of components enhance each other in the process of transverse current generation. So for the any closed contour in the plane ZX (for instance for the contour a-b-c-d, Fig.1) the summarizing of Hall electromotive forces takes place and as a result the circular current along this contour generates in accordance with the potentiality of steady electric field. For the contour a-b-c-d the next relation can be written:

$$\oint dl_i (\rho_{ik} j_k) = 0 \qquad (4)$$

Using the conception of symmetry of contour "a-b-c-d" it is possible to write the potentiality expression of (4) in integral form as:

$$\rho_{yx}\int j_x dy+\rho_{yy}\int j_y dy+\rho_{yz}\int j_z dy+\rho_{zy}\int j_y dz+\rho_{zz}\int j_z dz+Const=0 \qquad (5)$$

Here the integration along Y direction is in the limits from zero to b (b is a width of conductor), integration along Z direction is from zero (z -coordinate of interface is zero) to any point z (z is a variable of task). This equation is rather complicated for farther analysis. It is really to accept that the current densities j_x , j_y and j_z are the functions of the distance z from the interface to symmetrical with respect to interface contour lines "b-c", "a-d" . That is $j_x(z)$, $j_y(z)$. $j_z(z)$. So the equation (5) can be represented as:

$$\rho_{yx}j_x(z)b+\rho_{yy}j_y(z)b+\rho_{yz}j_z(z)b+\int\rho_{zy}j_y(z)dz+\int\rho_{zz}j_z(z)dz+Const=0 \qquad (6)$$

$$b\rho_{yx}\partial j_x/\partial z+b\rho_{yy}\partial j_y/\partial z+b\rho_{yz}\partial j_z/\partial z+\rho_{zy}j_y+\rho_{zz}j_z=0 \qquad (7)$$

Following the condition that the conductor is long in transport direction, following the current density continuity principle and using the condition that electric field component E_x is the same in the composite volume (it means that E_x is constant through conductor volume) :

$$\begin{aligned}
&\partial j_x/\partial x+\partial j_y/\partial y+\partial j_z/\partial z=0\\
&\partial E_x/\partial x=\rho_{xx}\partial j_x/\partial x+\rho_{xy}\partial j_y/\partial x+\rho_{xz}\partial j_z/\partial x=0\\
&\partial E_x/\partial z=\rho_{xx}\partial j_x/\partial z+\rho_{xy}\partial j_y/\partial z+\rho_{xz}\partial j_z/\partial z=0\\
&\partial E_x/\partial y=\rho_{xx}\partial j_x/\partial y+\rho_{xy}\partial j_y/\partial y+\rho_{xz}\partial j_z/\partial y=0
\end{aligned} \qquad (8)$$

it is possible to transform the equation (7) so to simplify it and to find the characteristic equation for current density under conditions have been accepted before:

$$\frac{dz}{-\dfrac{\rho_{yx}\rho_{xy}}{\rho_{xx}}+\rho_{yy}}=-\frac{b}{\rho_{zy}}\frac{dj_y}{j_y}=-\frac{b}{\rho_{zz}}\frac{dj_y}{j_z} \qquad (9)$$

accepting that at the interface of a composite (z = 0) the current density j_y is connected with j_x by simple relation j_y (z = 0) = - ρ_{yx}/ ρ_{yy} j_x (z = 0)(this relation follows from the demand of zero value of E_y in the plane of interface) the current density j_y may be represented as that having exponential law of dependence on coordinate z

$$j_y(z)=-(\rho_{yx}\sigma_{xx}/\rho_{yy})E_x\exp\left|\rho_{zy}zb^{-1}\left(\rho_{yx}\rho_{xy}\rho_{xx}^{-1}-\rho_{yy}\right)^{-1}\right| \qquad (10)$$

The more precisely connection between j_y (z = 0) and j_x (z = 0) is:

j_y (z = 0) = − (ρ_{yx}/ ρ_{yy}) $(1 + \rho_{yz}\rho_{zy}/\rho_{yy}\rho_{zz})^{-1}$ j_x (z = 0);

$\rho_{zy}\, j_y(z) = \rho_{zz}\, j_z(z).$

As the combination $\rho_{yz}\rho_{zy} / \rho_{yy}\rho_{zz}$ is of the order of unit in magnitude it is acceptable to ignore the combination keeping in mind that the real value of j_y $(z = 0)$ is a half of written that.
Here we neglect the inclusion in to transverse current density from electric field component E_z as the components ρ_{xy} are much higher of all others. Using the statement about the value of components ρ_{xy} comparatively to others mentioned above it is desirable to simplify the expression (10) via its linearization:

$$j_y(z) = -\frac{\rho_{yx}\sigma_{xx}E_x}{\rho_{yy}} \frac{1}{1+\dfrac{\rho_{zy}}{\rho_{yy}-\dfrac{\rho_{xy}\rho_{yx}}{\rho_{xx}}}\dfrac{z}{b}} \tag{11}$$

Here z coordinate has to be accepted in modulo, it indicates the distance to both symmetric "b-c" and "d-a" lines from interface of composite. Due to the chosen symmetry of picture these distances are the same. Eq. (11) shows that the transverse current is similar to Corbino circular current along closed circular contour but the difference is that the current under discussion is generated in the plane contour being parallel to magnetic field but Corbino current is generated in the plane contour being orthogonal to magnetic field. So using an integral form of equation for electric field circulation along closed contour of double strip conductor it is possible to get the expression for the magneto-stimulated current. Of course the transverse current is zero if the components of double strip conductor are the same in conductivity. Under the conditions of magnetic field absence the transverse current is zero too as $\rho_{xy} = 0$. Taking into account tensor relations between electric field and current density vector the connection for transport current density j_x distribution with an electric field component along transport direction E_x can be obtained. As a result the respective current density $j_x(z)$ may be represented as some function of task parameters:

$$j_x(z) = \left[\sigma_{xx} - \frac{\sigma_{xy}\sigma_{yx}}{\sigma_{yy}} - \frac{\sigma_{xx}}{\rho_{yy}\sigma_{yy}} \frac{1}{1+\dfrac{\rho_{zy}}{\rho_{yy}-\dfrac{\rho_{xy}\rho_{yx}}{\rho_{xx}}}\dfrac{z}{b}}\right] E_x \tag{12}$$

In this expression the involvement of carriers along z-direction under an action of external magnetic field takes place but the level of current generation along z-direction is very small in comparison with the scope of transverse and transport current. Actually Hall components of resistivity tensor in strong magnetic field are much higher of all other components and such an approximation is rather valid. Following equations the values of current j_y and transport current j_x are the functions of geometric parameters b and z. For the contour line having the ratio z/b being very large that the transverse current is small and j_x is determined as:

$$j_x = E_x / \rho_{xx}.$$

For the contour line having the small ratio z/b the transverse current has definite non-small magnitude and respectively the transport current density may be represented as

$$j_x = E_x / \rho_{xx}.\ (1 - \sigma_{xx} / \sigma_{yy})$$

That value is very small in magnitude because conductivity tensor components σ_{xx} and σ_{yy} are of the same order of magnitude.

If the independent variable of task is transport current density j_x the respective component of electric field E_x is very large.

Applying the procedure of averaging of transport current through the thickness of strip the final expression for the effective resistivity is the next:

$$\rho_{eff} = \frac{\rho_{xx}}{1-\left(\frac{RB}{\rho_{xx}}\right)^2 \frac{b}{t} \ln\left\{\left[1+\left(\frac{\rho_{xx}}{RB}\right)^{-2}\right]^{-1} \frac{t}{b}+1\right\}} \qquad (13)$$

Here the diagonal component of resistivity tensor ρ_{xx} may be not only constant as in former suggestions but this component is allowed to have more complicated behavior close to real that of polycrystalline aluminum when not very strong linear dependence of resistivity on magnetic field takes place. The eq.(13) is suitable to describe the current flow via composite conductor consisted of copper – anti-copper components similarly to present composite model. The polycrystalline structural action on to conductivity and resistivity tensors is to be taken into account. The anisotropy of conductivity of copper crystallite is usually averaged dynamically via polycrystalline copper conductor as random-inhomogeneous medium. The problem of charge transfer through copper polycrystalline conductor under the presence of strong magnetic field is discussed in our another Poster where the effective conductivity tensor is supposed in accordance with strong linear magnetic field dependence of resistance after Kapitca law. The dynamical averaging of conductivity of polycrystalline conductors have been analyzed in many articles. We however use more simple averaging on crystalline orientation only because more precise procedure leads to result that does not asymptotically differ hard from simplified geometrical averaging.

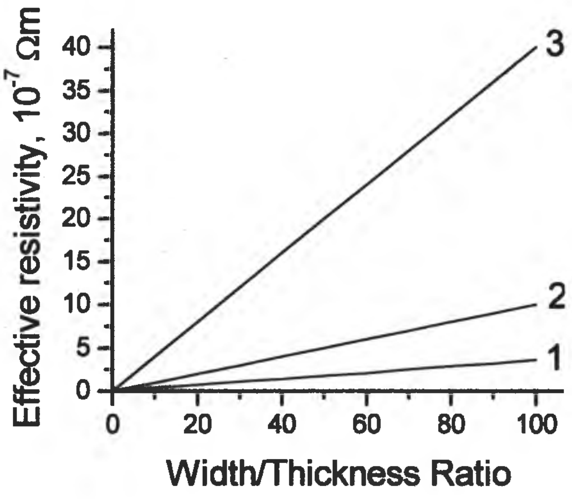

Fig.2. Effective resistance as a function of ratio b/t at different B, T: 3(1); 5(2); 10(3); $\rho_{xx} = 5{\cdot}10^{-11}\Omega{\cdot}\mathrm{m}$

On the base of Eq. (13) the effective resistivity of double strip model composite conductor under consideration have been calculated at different values of an external magnetic field oriented normally to interface of composite conductor, Fig.2. The linear dependence of effective resistivity on width/thickness ratio follows from the (13). In accordance with the Eq.(13) the effective resistivity is a function of ratio b/t where t is the thickness of strips. So for one limit case when $b/t \to 0$ the effective resistivity of composite conductor is equal to diagonal component of resistivity tensor ρ_{xx}. Actually, the function $\ln(t/b((RB/\rho)^2+1)^{-1} + 1)$ is much less of respective value $(RB/\rho)^{-2}\,(b/t)$ under condition $b/t \to 0$. Physically it means that the main part of current flows far from the conductor interface so in accordance with the principle of the minimum of entropy generation the system trends to such a state that ensures the minimal resistance at definite level of current flowing through cross section. The transport current density j_x is of the order of that for singular conductor under this field. Transverse current density j_y is zero near interface of conductor. The interface does not influence on to current and potential distribution.

Other limit case when $b/t \to \infty$ means that the thickness of every strip is so small in comparison with width that the process of shortening of transverse electric Hall field of every component with each other is a single process that can ensure the potentiality of steady electric field. At the condition $b/t \to \infty$ the effective resistivity trends to magnitude:

$$\rho_{eff} \to \rho_{xx}\,((RB/\rho_{xx})^2 + 1)$$

As a result the transverse Hall current is generated and the path of carriers along transport direction is occupated with transverse drift so that on the unit length on path along transport direction the charge carrier has time to drift in transverse direction and the transverse path is of ρ_{yx}/ρ_{xx} higher than the path in transport direction. As a result the collisions of charge carriers with crystal structure imperfections generate a respective resistivity being higher of that mentioned above. For this second limit case the effective resistivity trends to that of Corbino geometry conductor. For more realistic conditions of t/b ratio magnitude a some intermediate case of resistivity have to take place.

It is necessary to keep in mind that the magnitude of ratio RB/ρ_{xx} for pure aluminum conductor is of the order of 10^2 or less for the strong magnetic field (B is of the order of 7 –8 T, $R = 10^{-10}\ \Omega\cdot\mathrm{m}\cdot\mathrm{T}^{-1}$, $\rho_{xx} - 10^{-12}\ \Omega\cdot\mathrm{m}$). Respectively for all diapason of relevant values of ratio t/b the character of current flow very close to Corbino will take place.
Essentially that for the case of real composite double strip conductor made of aluminum and copper the more less linear law of resistivity increase will take place because the strong dependence of diagonal component on magnetic field for copper will restrict the RB/ρ ratio. However the contribution to the total process of aluminum component seems to be dominant. This task should be analyzed in future.

4. Conclusion

The expression obtained for effective resistivity of double-strip composite conductor placed in magnetic field is rather idealized that as it operates with model composite material. However the approaches developed for this task here seems to be rather reliable as these allow to construct a real physical picture under the presence of opposite types of magneto-stimulated anisotropy of conductivity and resistivity tensors. The father steps in the study of this problem should be done in the direction of understanding the averaging procedure along transverse direction, the analysis of non-symmetric task on geometry and on conductive properties of components. The last thesis is adequate to the situation of real double-strip composite conductor consisted of aluminum and copper components.
At low temperatures the total current value is more suitable for controlling the charge transfer process in highly conducting materials. So it would be reasonable to analyze the electric field and current distribution picture from this point of view namely via controlling the total current of energy supply via conductor cross section instead of controlling the elelctric field. That means that integral current value of energy supply source is a basic component of task that has to be attracted for obtaining the electric field distribution and energy dissipation that determines the effective resistance.

References

[1] P. W. Eckels, N. C. Iver, A. Paterson, A. T. Male, J. H. Parker, Jr. And J. W. Coltman, "Magnetoresistance: the Hall effect in composite aluminum cruoconductors", Cryogenics, Vol. 29, p.748, 1989.

[2] F. R. Fickett and C. A. Tompson, ""anomalous magnetoresistance in Al/Al-Alloy composite conductors", Advances in Cryogenic Engineering (Materials), Vol. 36, Edited by R. P.Reed and F. R. Fickett, Plenum Press, New York, 1990, pp.671 – 677.

[3] P. W. Eckels and J. H. Parker, Magnetoresistance in composite conductorsV. Advances in Cryogenic Engineering (Materials), Vol. 36, Edited by R. P. Reed and F. R. Fickett, Plenum Press, New York, 1990, pp. 655 – 662.

[4] H. Kaneko, and N. Yanagi, "Enhancement of magnetoresistance due to Hall current in aluminum-copper composites", Cryogenics, Vol. 32, No. 12, 1992, p. 1114.

[5] H. Kaneko, "Hall currents in aluminum-copper composites", Advances in Cryogenic Engineering (Materials), Vol. 40A, Edited by R. P. Reed and F. R. Fickett, T. Summers and M. Steig, Plenum Press, New York, 1994, pp. 451-458.

[6] Y. Yanagi et al., "Experimental observation of anomalous magnetoresistivity in 10-20kA class aluminum stabilized superconductors for the Large Helical Device", Advances in Cryogenic Engineering (Materials), Vol. 40A, Edited by R. P.Reed and F. R. Fickett, T. Summers and M. Steig, Plenum Press, New York, 1994, pp. 459-468.

[7] S. Imagawa et al., "Analysis of anomalous resistivity in an aluminum stabilized superconductor for Large Helical Device", Advances in Cryogenic Engineering (Materials), Vol. 40A, Edited by R. P. Reed and F. R. Fickett, T. Summers and M. Steig, Plenum Press, New York, 1994, pp. 469-477.

[8] F. R Fickett, "Magnetoresistance of very pure polycrystalline aluminum", Physical Review B, Vol.3, No. 6, March 1971, p.1941.

[9] R. J. Corruccini, "The electrical properties of aluminum for cryogenic electromagnets", NBS TN 218, 1964.

[10] B. Krevet et al., "Transverse magnetoresistance and its temperature dependence for high-purity polycrystalline aluminum", Journal of Applied Physics, Vol. 47, No. 8, Aug. 1976, pp. 3656-3669.

[11] B. B. Boiko, V. R. Sobol, O. N. Mazurenko, and A. A. Drozd, "Magnetosensitive conductivity of aluminum and the advantage of Corbino geometry", Advances in Cryogenic Engineering (Materials), Vol. 42, Edited by L. T. Summers, Plenum Press, New York, Vol. 42, 1996, pp. 1063-1069.