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U. BALU BALACHANDRAN, DONALD G. GUBSER, K. TED HARTWIG,
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Edited by

U. Balu Balachandran
Argonne National Laboratory
Argonne, Illinois

Donald G. Gubser
Naval Research Laboratory
Washington, D.C.

K. Ted Hartwig
Texas A&M University
College Station, Texas

Richard P. Reed
Cryogenic Materials, Inc.
Boulder, Colorado

William H. Warnes
Oregon State University
Corvallis, Oregon

Victoria A. Bardos
Synchrony
Bordentown, New Jersey

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ON EXCESSIVE RESISTANCE AND HEAT GENERATION OF ALUMINUM CONDUCTOR DUE TO MAGNETIC FIELD INHOMOGENEITY

V.R.Sobol, O.N.Mazurenko, and A.A.Drozd

Institute of Solid State and Semiconductor Physics NASB
Minsk 220072, Belarus

ABSTRACT

One of major problem of cryogenic devices generating high magnetic field is an enhancement of magnetoresistance and respective heat generation due to inhomogeneity of composite conductor in the place of its interface. It should be mentioned that there is an inhomogeneity of magnetic field through device's coils because the inner and outer coils are at different magnitudes of magnetic field. Usually a contribution of magnetic field inhomogeneity into resistance increase is neglected. A paper proposed here is to show that at cryogenic temperatures the amount of excessive resistance connected with magnetic field inhomogeneity may be comparable in magnitude with traditional enhancement of resistance due to interface of composite conductors. The testing is done by measurement of potential distribution over aluminum conductors being under inhomogeneous magnetic field action at 4.2 K. The magnetic field gradient up to 35 kOe/cm is directed along current flow. A calculation of nonuniform current distribution and enhancement of resistance due to decrease of effective conductor cross section is made. A correlation of experimental data with calculation is established and discussed.

INTRODUCTION

The superconductive cable stabilization at thermal fluctuations is based on use of normal metal components for example Al, Cu. These materials have a high thermal conductivity to decrease local heating of cable. Excessive resistivity of composite materials Al - Cu, Al (pure) - Al (dirty) etc. being stabilizers is widely investigated because the inhomogeneous region of interface in junction of these materials having different conductivity and Hall coefficient stimulates an appearance of additional current and excessive energy generation.[1-6] However an excessive resistance calculations on the base of parallel circuit model using measured core magnetoresistivity, or on the base of pure aluminum parameters have a definite (about 25%)

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discrepancy with data of experiment. In other words the experimental data on excessive magnetoresistance are higher of analytical ones.

In this paper we should like to attract an attention to additional possible reason of excessive resistivity. This is a magnetic field itself or its inhomogeneity. It is necessary to say that in cryogenic experiment a magnetic field is usually generated with system of limited dimentions because a volume of both cryostat and its helium cavity is restricted. As a result a generated magnetic field has a definite degree of inhomogeneity. The inhomogeneity can affect on results of measurements.[7-9] On the other hand in any solenoidal system generating magnetic field the inner and outer coils are in magnetic field of different magnitude. So there is a magnetic field gradient along cable. In such system a magnetic field has two components: an axial one and a radial one. An axial component decreses along inner layer being maximal in the center and minimal at the edge. For the long solenoid (the length of system is much higher of its outer diameter) the decrease of axial component is about 50%. The radial component increases from a center to an edge and is more difficult for estimation. In the long system the radial component also may be estimated and its value is about 25% of inner axial component. The outer layers of solenoid are in the magnetic field of less amplitude and the magnetic field spatial variation is less. Here we investigate the influence of magnetic field inhomogeneity on current flow properties of metal stabilizing conductors on the subject of pure aluminum.

EXPERIMENTAL PROCEDURE AND ANALYTICAL APPROACH

High pure aluminum plate conductors have been used for experiment. The residual resistance ratio RRR of material was about 10000. Measurements have been done in helium cryostat at temperature 4.2 K. The magnetic field inhomogeneity was modeled by the method of curving the current lines for sample situated in strong magnetic field.[8] The curving of current lines and respective spatial variation of local normal component of external magnetic field along current direction was made by making of necessary profile of investigated sample. In other words the plate samples were bent along transport direction in accordance with such law that an inhomogeneity of normal component of field with fixed gradient has been ensured along all length of sample under measurement. Magnetic field gradients of 0 ÷ 35 kOe/cm have been realized. The attention was payed to the phenomena of current skinning that is to the current density redistribution and its role in conductor resistance. It is followed from the common relations that the current skinning denotes the enlarge of current density by one of side parallel to magnetic field and the decrease of current density by another side. In this case an effective cross section decreases. Such fact leads to increase of resistance because it is a parameter being reversely proportional to effective cross section. Resistance is a characteristic describing ohmic losses and is an integral one. On the other hand the current density redistribution and a growth of resistance may be regarded as a consequence of appearance of additional current being dissipative. In this approach a local resistivity is constant along transverse (Hall) direction and is varied along current flow in accordance with experimental conditions.

The research has been done by the method of comparison of electric field potential and its spatial dependence on coordinates for two conditions: under action of homogeneous and inhomogeneous magnetic field, the magnetic field gradient being controlled. Analytical study has been done on the base of model representations of current flow in conductor having isotropic dispersion law and electron scattering processes described with relaxation time τ approximation. Partly this problem has been discussed in our previous work.[8] Some elements of the influence of dispersion law anisotropy onto current redistribution and excessive

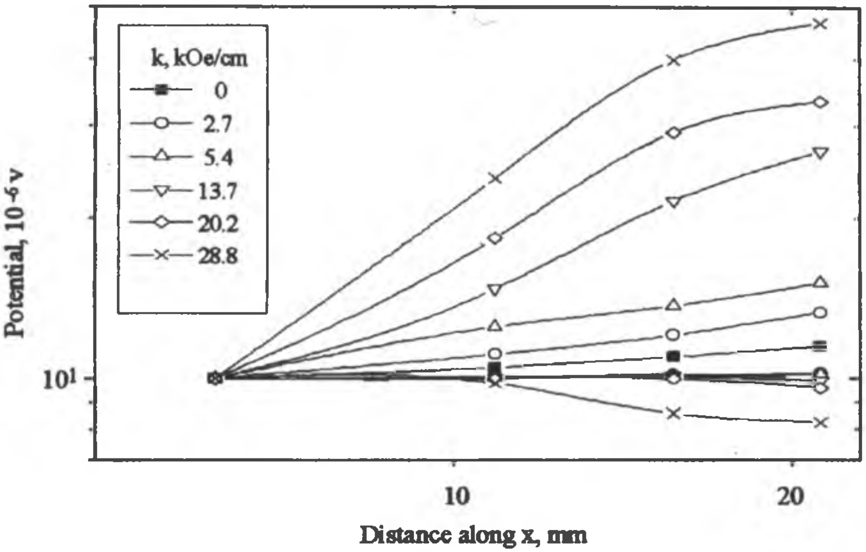

Figure 1. Potential distribution along x for homogeneous (solid squares) and inhomogeneous (open symbols) conditions. Data for strong side - over homogeneous curve; data for weak side - under homogeneous curve. Magnetic field gradients are 0 ÷ 28.8 kOe/cm. Homogeneous field is 55 kOe.

resistance in inhomogeneous magnetic field are discussed here on the base of principles of the charge discontinuity and anisotropic connection between current density and stationary electric field.

EXPERIMENTAL DATA AND DISCUSSION

Stimulated by nonuniform magnetic field a current density redistribution is known to generate the inhomogeneous potential distribution through sample cross section. On opposite sides of sample the potential levels are differ more significantly. There is a potential of high level on one of sides, we shall call it as a strong side. On the opposite side the potential and its difference are weak, we shall call this side as a weak one. High current density takes place by the strong side and small current density is localized near weak side. Fig.1 represents the potential and its dependence on coordinate in transport direction for magnetic field gradients 0 ÷ 28.8 kOe/cm under magnetic field 55 kOe. Potential dependences are depicted for both sides of conductor. For homogeneous field (k = 0) these dependences coincide. For inhomogeneous field the results for strong side lie higher of homogeneous curve and for weak side the curves lie lower of homogeneous curve.

It is seen that the inhomogeneity stimulates the divergence between potential behaviour on strong and weak sides. Nonsymmetry of characteristics on strong and weak sides with respect to homogeneous distribution is observed. In other words the difference between a potential on strong side and a homogeneous potential is higher of respective difference between signal on weak side and a homogeneous signal. This nonsymmetry is connected with nonlinearity of potential and current density as a function of transverse Hall coordinate y.

The ratio of excessive resistance R^e to homogeneous resistance R has been calculated on the base of relation

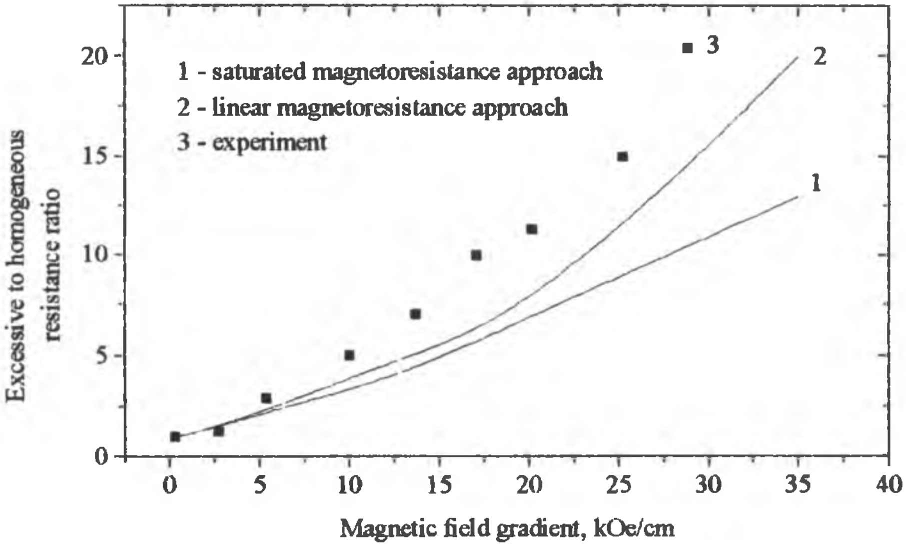

Figure 2. The ratio of excessive resistance to homogeneous resistance R^e/R as a function of magnetic field gradient. Square symbols - experiment, solid lines - theory.

$$\frac{R^e}{R}=\frac{\Delta\varphi_1+\Delta\varphi_2-\Delta\varphi}{\Delta\varphi} \tag{1}$$

Here $\Delta\varphi_1$ and $\Delta\varphi_2$ is a potential differency for strong and weak side respectively, $\Delta\varphi$ is a potential differency under an action of homogeneous magnetic field. The homogeneous field and the magnitude of inhomogeneous field at one of edge of sample (at coordinate $x = 0$) are the same. In Fig.2 a ratio R^e/R is expressed as a function of magnetic field gradient. This method of determination of an excessive resistance is based on approximation of equivalent homogeneous current distribution in the conductor of less cross section. In such approach the resistivity is supposed to be the same through all volume and the only reason of resistance increase is a decrease of cross section. In limit case of zero inhomogeneity ($k \to 0$) a ratio $R^e/R \to 1$ because potential on weak and strong sides becomes symmetric with respect to homogeneous action as experiment has shown.

The calculated data for aluminum conductor have been obtained at different approaches. Excessive resistivity stimulated by magnetic field inhomogeneity has been analysed using a potential distribution in inhomogeneous field. The ratio of excessive resistance R_e to homogeneous resistance R has been analysed on the base of relation

$$\frac{R_e}{R}=\frac{S\int \rho j^2 dS}{\rho I^2} \tag{2}$$

Here j is a current density, S is a cross section of conductor, I is a total current through cross section, ρ is a resistivity.

For free electron approximation a current density dependence on coordinates is

$$j_x = \frac{I\beta'}{t}\frac{\exp(\beta' y)}{\exp(\beta' b)-1}; \quad j_y = 0 \tag{3}$$

Here t is a sample thickness, $\beta' = \frac{e\tau}{mc}\frac{dH}{dx}$, e and m are the charge and the mass of electron, c is the light velosity.

For model case of closed Fermi surface and τ-approximation we have

$$\frac{R_e}{R} = b\frac{\rho'_{yx}}{\rho_{xx}}\frac{\exp(2b\rho'_{yx}/\rho_{xx})-1}{2\left[\exp\left(b\rho'_{yx}/\rho_{xx}\right)-1\right]^2} \tag{4}$$

Here ρ'_{yx} is a derivative of ρ_{yx} with respect to coordinate along magnetic field variation, b is a width of sample.

Eq.(4) is followed from Eq.(3) where a parameter β' is replaced by ρ'_{yx}/ρ_{xx}. The account of anisotropy of aluminum dispersion law demand to use a transverse magnetoresistance $\rho_{xx} = \rho_s(1+\lambda H)$ where ρ_s is a saturated part of magnetoresistance. The second term in brackets corresponds to a field dependence of resistivity. For weak non-saturation when the parameter λ is so small that λH is much smaller of unit we can ignore a linear term. The current redistribution determines the resistance ratio in accordance with Eq.(4).

The account of linear dependence of resistance on magnetic field stimulates the appearance of transverse current

$$\begin{aligned} j_x &= \frac{I\alpha}{t}\frac{\exp(\alpha y)}{\exp(\alpha b)-1}\left[1-\frac{1}{2}\frac{(\lambda kb)^2}{(1+\lambda kx)^2}\right]-\frac{(\lambda k)^2}{(1+\lambda kx)^2}\frac{I}{t}(y-b) \\ j_y &= \frac{I}{t}\frac{\alpha\lambda k}{(1+\lambda kx)^2}(y-b) \end{aligned} \tag{5}$$

Here $k = dH/dx$, $\alpha = \rho'_{yx}/\rho_{xx}$.

The resistance ratio for current distribution (5) is

$$\frac{R'}{R} = \frac{b\alpha}{2}\frac{\exp(2\alpha b)-1}{\left[\exp(\alpha b)-1\right]^2}\left[1-\frac{(\lambda kb)^2}{(1+\lambda kx)^2}\right]+\frac{1}{3}(\alpha b)^2\frac{(\lambda kb)^2}{(1+\lambda kx)^2} \tag{6}$$

Eq.(4) have been obtained in approximation of ρ'_{yx}/ρ_{xx} = const. This denotes that two different variants of parameters and their relations may be analysed in experiment on excessive magnetoresistance in conductors under inhomogeneous magnetic field action. The first variant corresponds to conductors having ρ_{xx} = const. In this case a derivative dH/dx also is constant because $\rho_{yx} = RH$ (here R is Hall coefficient). Second variant of consideration is suitable for conductors having $\rho_{xx} \sim H$. For such conductors Eq.(4) describes an excessive inhomogeneous magnetoresistance when a law of magnetic field spatial variation is an exponential one. Actually for this case $1/H(dH/dx)$ is constant if magnetic field dependence of ρ_{xx} is linear. The first variant is more suitable for aluminum and conductors of similar type. So an inhomogeneous magnetic field having linear law of spatial variation $H = H_0 + kx$ may be used to study the peculiarities of current skinning and inhomogeneous excessive magnetoresistance in experiment with further theoretical analysis and correlations between experimental conditions and properties of material under investigation. The second variant of approach describes a current redistribution in conductor having Fermi surface resembling a copper that. In polycrystalline copper samples the presence of open electron orbits generates a strong linear dependence of ρ_{xx} on magnetic field and Eq.(4) describes an excessive resistance ratio at exponential type of inhomogeneity. Eq.(5) corresponds to intermediate case when a magnetoresistance has a saturating part and a weak nonsaturation. This approximation is the most real for aluminum conductors. Calculations having been done for polycrystalline aluminum samples in accordance with approaches for saturation and weak nonsaturation show a good correspondence with experiment.

The temperature affection on the phenomenon of current redistribution and excessive resistance generation must be determined by behaviour of ρ_{yx} and ρ_{xx}. Hall coefficient of indicated metals does not depend on temperature in helium range. However a diagonal component of resistivity tensor ρ_{xx} is a function of temperature. As a result the parameter α in Eq.(4-6) decreases at temperatures 20 - 30 K. So there takes place a tendency to diminishing the degree of current skinning and to decrease of excessive inhomogeneous resistance at hydrogen and neon temperatures.

CONCLUSION

Current density redistribution or steady current skinning takes place in metal conductors when transverse magnetic field being inhomogeneous along current flow is applied.

Steady current skinning stimulates an excessive magnetoresistance due to a decrease of effective cross section of conductor.

Experimental and theoretical data show that for aluminum conductor the magnetoresistance increase stimulated by inhomogeneous magnetic field is rather large already at low degrees of inhomogeneity.

Analytical consideration shows that the level of excessive resistance is determined with magnitude of magnetic field and a type of its inhomogeneity. The purity of metal, its type of dispersion law and temperature are essential too.

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