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Domination of Cyclic Monotone (s, t)-Graphs

A. A. Chernyak

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Abstract. The conjecture of zero domination of 0-cyclic monotone graphs is proved (an r-cyclic graph is a cyclic monotone (s,t)-graph exactly r minimal paths of which have cycles). As a corollary, a formula for computing the reliability of an arbitrary 0-cyclic monotone graph is obtained. It is proved that the problem of determining the domination in the class of r-cyclic monotone graphs is $\#P$-complete for any fixed integer $r \geq 1$.

Key words: domination of graphs, cyclic monotone graphs, stochastic graphs, reliability, $\#P$-completeness.

1. Introduction

Domination theory, initiated in [1], gave an impetus to the development of combinatorial methods for solving network reliability problems. It served as a basis for essentially new algorithms for computing the reliability of stochastic graphs. Later on, in [2–8], the range of applicability of this theory was expanded and the connection of domination parameters with matroid invariants and reliability polynomials was established.

However, the diversity of network models of reliability called for extending domination theory to stochastic graphs with more complex logical functions of their vertices. For instance, acyclic monotone (s,t)-graphs, first introduced in [9], admit arbitrary monotone Boolean functions at vertices (the only kind of vertex functions allowed in ordinary stochastic graphs are elementary disjunctions; see Sec. 2). Acyclic monotone (s,t)-graphs include as particular cases the classical models of multi-terminal reliability of directed graphs [10–13]. In [14], the notion of local domination was introduced and a formula effectively computing the domination of acyclic monotone (s,t)-graphs in terms of local dominations of its vertices was obtained. This revealed the common combinatorial nature of earlier results [1, 10, 11, 15, 16] concerning the domination of directed graphs without cycles. But the problem of determining the domination of cyclic monotone (s,t)-graphs remained open.

This article fills this gap. Namely, Theorem 1 proves the conjecture [7, 14] about zero domination of 0-cyclic monotone graphs (an *r-cyclic monotone graph* is a cyclic monotone (s,t)-graph in which exactly r minimal paths have cycles). As a corollary, a formula for computing the reliability of an arbitrary 0-cyclic monotone graph (Corollary 1) is obtained. It is proved that the problem of computing the domination in the class of r-cyclic monotone graphs is $\#P$-complete for any fixed integer $r \geq 1$ (Theorem 2). It should be mentioned that the results concerning cyclic graphs from [1, 5, 10, 11, 17] directly follow from Theorem 1 (two of these results are given in Corollaries 2, 3).

2. Monotone (s,t)-graphs: preliminary definitions

All the graphs considered below are assumed to be directed. Let VG, DG be the sets of vertices and edges of a graph G, respectively. By (u,w) we denote the edge directed from u to w; by $D^+(v,G)$ ($D^-(v,G)$), the set of edges of the graph G directed to the vertex v (from the vertex v, respectively). A sequence of edges $e_i = (v_i, v_{i+1})$, $i = 1, \dots, n$, is called a *simple chain* (or a (v_1, v_{n+1})-*chain*) if all the vertices $v_1, \dots, v_{n+1}$ are pairwise distinct. If only the vertices $v_1, \dots, v_n$ are pairwise distinct, whereas $v_1 = v_{n+1}$, then this sequence of edges is called a *cycle*. If a graph G contains at least one cycle, it is called *cyclic*; otherwise, G is called *acyclic*; G is called an (s,t)-*graph* if

$$s, t \in VG, \quad D^-(t,G) = D^+(s,G) = \varnothing, \quad D^-(v,G) \neq \varnothing, \quad D^+(v,G) \neq \varnothing$$

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for any vertex $v \in VG \setminus \{s, t\}$; the vertices s and t are called the *input* and *output poles* of the graph G, respectively.

Now let us define a monotone (s,t)-graph G. With each vertex $v \neq s$ of the graph G, we associate the collection $\mathrm{th}(v, G)$ of *threshold sets* of the vertex v in G, the subsets of $D^+(v, G)$ with the following properties:

(a) none of the threshold sets is contained in another;

(b) each element from $D^+(v, G)$ belongs to a certain threshold set from $\mathrm{th}(v, G)$.

If all threshold sets from $\mathrm{th}(v, G)$ are of the same cardinality k and each k-element subset in $D^+(v, G)$ lies in $\mathrm{th}(v, G)$, then the set $\mathrm{th}(v, G)$ is called *symmetric* and the number k is called its *threshold number.*

An (s,t)-graph H is called a *minimal path* of a graph G if it is a subgraph of G such that for each vertex $v \in VH \setminus \{s\}$ the set $D^+(v, H)$ is a threshold set of the vertex v in G.

Also, we assume that to each edge e of the graph G the indicator function x_e is assigned that takes the value 1 with a given probability p_e. The *reliability* of the graph G is defined as the probability that G contains at least one minimal path such that all the indicator functions of its edges take the value 1.

A graph G thus defined is said to be a *monotone (s,t)-graph.* The definition of an r-cyclic monotone graph was given in Sec. 1.

A subgraph H of a monotone (s,t)-graph G is called a *regular subgraph* if it can be represented as the union of minimal paths of the graph G. In addition, we assume that the collection $\mathrm{th}(v, H)$ of the threshold sets of the vertex v in the graph H consists exactly of the threshold sets from $\mathrm{th}(v, G)$ that are contained in the set $D^+(v, H)$ (in other words, $\mathrm{th}(v, H)$ is induced by the set $\mathrm{th}(v, G)$).

Let us consider a number of particular cases of monotone (s,t)-graphs.

(1) A 2-terminal stochastic graph [1, 12]. It is an (s,t)-graph G whose reliability is defined as the probability that G contains at least one (s,t)-chain such that the indicator functions of all its edges take the value 1.

In terms of our definitions, G is a monotone (s,t)-graph in which the set $\mathrm{th}(v, G)$ for each vertex $v \neq s$ is symmetric with threshold number 1, and the minimal paths of G are its (s,t)-chains.

(2) A source-K-terminal stochastic graph [10, 12]. Suppose that a graph H has one input pole s and a set $K = \{t_1, \dots, t_r\}$ of output poles. The reliability of H is defined as the probability that there is at least one K-tree such that the indicator functions of all its edges take the value 1 (a *K-tree* is a tree with root s whose hanging vertices all belong to the set K; a *hanging vertex* is a vertex adjacent to a single edge, the edge being directed to this vertex).

Let us add to H a new vertex t and r new edges (t_i, t), $i = 1, \dots, r$, whose indicator functions take the value 1 with probability 1. Denote the new graph by G. Then, by our definitions, G is a monotone (s,t)-graph in which $\mathrm{th}(t, G)$ is a one-element set with threshold number r, and $\mathrm{th}(v, G)$ for each vertex $v \neq s, t$ is a symmetric set with threshold number 1.

(3) An acyclic monotone (s,t)-graph [8, 13–15]. Let us assign to each vertex $v \neq s$ of an (s,t)-graph G the vertex function $g(v, G) = g(z_1, \dots, z_n)$ defined as the disjunction of $|\mathrm{th}(v, G)|$ elementary conjunctions $z_{i_1}, \dots, z_{i_k}$ bijectively corresponding to the threshold sets $\{e_{i_1}, \dots, e_{i_k}\} \subseteq D^+(v, G)$ from $\mathrm{th}(v, G)$. In addition, set $g(s, G) \equiv 1$. Also, the output signal y_v is defined to be equal to the value of the vertex function $g(v, G)$. The Boolean variable z_i is set to be equal to 1 if and only if the indicator function x_i of the edge $e_i = (w_i, v)$ takes the value 1 and $y_{w_i} = 1$. Then the reliability of G is defined as the probability of the event $y_t = 1$.

3. Domination of cyclic graphs

Let G be a monotone (s,t)-graph. A *formation* of the graph G is the subset of its minimal paths whose union is G. The minimal paths here are called *components of the formation.* A formation is said to be *even* or *odd* depending on the parity of the number of its components. The *domination* $d(G)$ of the graph G is the difference between the numbers of odd and even formations of the graph G. If G is not a regular graph, then by definition, $d(G) = 0$.

A *local formation* of a vertex $v \neq s$ in G is a subset of threshold sets from $\mathrm{th}(v, G)$ whose union is $D^+(v, G)$. The difference $d(v, G)$ between the numbers of odd and even local formations of a vertex $v \neq s$ in G is called the *local domination* of the vertex v. It is assumed that $d(s, G) = 1$.

A set of minimal paths of the graph G is called a *covering of a subgraph* H if each edge of H belongs to at least one minimal path from this set.

Lemma 1. *Let G be a regular 0-cyclic monotone (s,t)-graph. Then there exists a pair (L,F) such that L is a cycle in G and F is a minimal path in G that does not belong to any minimal (by inclusion) covering of the cycle L.*

Proof. We shall say that an edge $e=(w,v)$ in the graph G is *irreducible* if it is contained in each threshold set of the vertex v; otherwise, e will be called a *reducible* edge.

First we suppose that there exist a cycle L and a minimal path F such that F does not contain any reducible edges of the graph G belonging to L. Also, we suppose that $\mathcal{P}$ is a minimal covering of the cycle L that belongs to F. Then there exists an edge $e_1=(u_1,u_2)$ in $L\cap DF$ that does not belong to any minimal path from $\mathcal{P}\setminus\{F\}$.

Set $L=e_1,\ldots,e_n$, where $e_i=(u_i,u_{i+1})$, $i=1,\ldots,n$, $u_1=u_{n+1}$. Let k be the smallest i such that $e_i\notin DF$ (this k exists by acyclicity of F). Then e_k belongs to a certain minimal path A from $\mathcal{P}$ and the edges $e_1,\ldots,e_{k-1}$ are irreducible. The definitions of irreducibility and minimal path yield the implication

$$(u_{i+1}\in VA)\implies(e_i\in DA).$$

It follows that $e_1\in DA$. A contradiction. Now it remains to prove the existence of a pair (L,F) such that the minimal path F does not contain reducible edges of the cycle L.

To prove this fact, we shall need the following procedure, denoted by $\mathrm{Proc}(t,\mathcal{L})$, for constructing a subgraph of the graph G in accordance with a certain rule of choice $\mathcal{L}$:

Step 1. Declare the vertex s to be boundary. Set $VH=\varnothing$, $DH=\varnothing$, $u=t$ and proceed to Step 2.

Step 2. Declare the vertex u to be boundary. Choose a threshold set $T(u)\in\mathrm{th}(u,G)$ according to the rule $\mathcal{L}$ and set

$$VH=VH\cup\{u\}\cup\{\text{initial vertices of the edges of the threshold set } T(u)\},\ DH=DH\cup T(u).$$

Declare the initial vertices of the edges from $T(u)$ to be potential vertices. Proceed to Step 3.

Step 3. Choose an arbitrary vertex u that is potential but not boundary. If there are no such vertices, the procedure is complete; otherwise, pass to Step 2.

Now let C be an arbitrary cycle in G. Since G is coherent, there exists an (s,t)-chain $R=q_1,\ldots,q_n$ that has common edges with C. Let us take the two vertices v_r and v_l in the set $VR\cap VC$ closest to the vertices s and t, respectively, counting along the chain R. Denote by C_1 the sequence of edges of the cycle C that constitutes a (v_r,v_l)-chain; let q_{r-1} and q_l be the edges of the chain R directed to the vertex v_r and from the vertex v_l, respectively. Obviously, $P=q_1,\ldots,q_{r-1},\ C_1,\ q_l,\ldots,q_n$ is an (s,t)-chain in G and $VP\cap VC=VC_1$. It follows that the implication

$$(u\in VC\cap VP,\ u\neq v_r,\ vu \text{ is an edge of the cycle } C)\implies(vu\in P\cap C)\tag{1}$$

is true.

Denote by A the subgraph constructed by the procedure $\mathrm{Proc}(t,\mathcal{L}_1)$ according to the following rule of choice $\mathcal{L}_1$:

if $u\in VP$, then $T(u)\cap P\neq\varnothing$;
if $u\in VC\setminus VP$, then $T(u)\cap C\neq\varnothing$;
in the remaining cases the choice of the threshold set $T(u)$ is arbitrary.

By the definition of the rule $\mathcal{L}_1$, we have $P\subseteq DA$. Therefore, A is an (s,t)-subgraph. Hence, by the definition of $\mathrm{Proc}(t,\mathcal{L})$, A is a minimal path.

Now denote by B the subgraph constructed by the procedure $\mathrm{Proc}(t,\mathcal{L}_2)$ according to the following rule of choice $\mathcal{L}_2$ (here $Q=q_1,\ldots,q_{r-1}$):

if $|\mathrm{th}(u,G)|>1$, $u\in VA$, $u\notin VQ$, then $T(u)\neq D^+(u,A)$;
if vu is a reducible edge of the cycle C, then $vu\notin T(u)$;
if $u\in VQ$, then $T(u)\subseteq DA$;
in the remaining cases the choice of the threshold set $T(u)$ is arbitrary.

The first two conditions of the rule $\mathcal{L}_2$ are consistent. Indeed, if vu is a reducible edge of the cycle C, $u \in VA$, $u \notin VQ$, then in view of (1) and the rule of choice $\mathcal{L}_1$, we have $vu \in DA$, i.e., we can always find a threshold set $T(u)$ which does not contain vu and at the same time does not coincide with $D^+(u, A)$.

If B is a minimal path, then the pair (C, B) is the desired one, because B does not contain reducible edges of the cycle C. Suppose that this is not true. Then B does not contain (s,t)-chains, and so $VB \cap VQ = \varnothing$. Hence the implication

$$(\mathrm{th}(u, G) > 1,\ u \in VB \cap VA) \implies (D^+(u, B) \neq D^+(u, A))$$

is true (see the definition of the rule of choice $\mathcal{L}_2$). Therefore, to each vertex $u \in B$, we can assign an edge $e(u)$ as follows:

if $|\,\mathrm{th}(u, G)| > 1$, then $e(u) \in D^+(u, B) \setminus D^+(u, A)$;
if $|\,\mathrm{th}(u, G)| = 1$, then $e(u) \in D^+(u, B)$.

Denote by R the subgraph of the graph B with the set of edges $\{e(u) : u \in VB\}$. Let N be a cycle in the graph R (such a cycle exists, because R does not contain (s,t)-chains and $D^+(u, R) \neq \varnothing$ for all $u \in VR$). If $e(u) = wu$ is a reducible edge of the cycle N, then $|\,\mathrm{th}(u, G)| > 1$, and so $e(u) \notin DA$. Therefore, (N, A) is the desired pair. This completes the proof of Lemma 1. □

If $\mathcal{P}$ a set of finite subsets, then we shall denote by $\mathrm{od}(\mathcal{P})$ and $\mathrm{ev}(\mathcal{P})$ the numbers of elements from $\mathcal{P}$ of odd and even cardinality, respectively.

Theorem 1. *Let G be a regular 0-cyclic monotone (s,t)-graph. Then $d(G) = 0$.*

Proof. Suppose that 0-cyclic monotone graphs with nonzero domination exist. Among these graphs we choose the graph G with the smallest number of edges. By Lemma 1, there exists a pair (L, F) such that L is a cycle in G and F is a minimal path not belonging to any minimal covering of the cycle L. Denote by $\mathcal{P}_1$ the set of formations of the graph G that do not contain F, by $\mathcal{P}_2$ the set of formations of G obtained by adding to each formation from $\mathcal{P}_1$ the minimal path F, and by $\mathcal{P}_3$ the set of all other formations of the graph G (each of them contains F). Obviously, $\mathrm{od}(\mathcal{P}_1) = \mathrm{ev}(\mathcal{P}_2)$, $\mathrm{od}(\mathcal{P}_2) = \mathrm{ev}(\mathcal{P}_1)$.

Now let $\mathcal{E}$ be the set of subsets of minimal paths obtained by removing the minimal path F from each formation of $\mathcal{P}_3$. Then each element from $\mathcal{E}$ defines a certain proper subgraph of G (by the definition of $\mathcal{P}_3$). Let $\{H_i : i = 1, \dots, n\}$ be the set of such subgraphs. By the choice of the pair (L, F), each H_i contains the cycle L. But then by the induction conjecture, $d(H_i) = 0$, $i = 1, \dots, n$. Further, we have

$$d(G) = \sum_{i=1}^{3} \big(\mathrm{od}(\mathcal{P}_i) - \mathrm{ev}(\mathcal{P}_i)\big) = \mathrm{od}(\mathcal{P}_3) - \mathrm{ev}(\mathcal{P}_3) = -\big(\mathrm{od}(\mathcal{E}) - \mathrm{ev}(\mathcal{E})\big) = -\sum_{i=1}^{n} d(H_i) = 0.$$

A contradiction. The proof is complete. □

Denote by $\Pr(H)$ the probability that the indicator functions of all edges of the graph H take the value 1.

The following statements are immediate consequences of Theorem 1 and [14, Theorem 1].

Corollary 1. *Suppose that G is a 0-cyclic monotone (s,t)-graph, $R(G)$ is the set of its regular acyclic subgraphs, and $\mathrm{Rel}(G)$ is the reliability of the graph G. Then*

$$\mathrm{Rel}(G) = \sum_{H \in R(G)} d(H) \cdot \Pr(H),$$

where $d(H) = \prod_{v \in VH} d(v, H)$.

Corollary 2 (see [1]). *The domination of a cyclic 2-terminal graph is 0.*

Corollary 3 (see [5, 10, 17]). *The domination of a cyclic source-K-terminal graph is* 0.

Our treatment of the notion of $\#P$-completeness of enumeration problems follows the papers [18, 19].

Let us give one more definition. Suppose that $\mathcal{E} = \{e_1, \dots, e_n\}$ is a set of elements; $\mathcal{P} = \{P_1, \dots, P_m\}$ is a set of subsets (called *minimal paths*) of the set $\mathcal{E}$ such that $P_i \not\subseteq P_j$ for any $i \neq j$. The pair $[\mathcal{E}, \mathcal{P}]$ is called a *binary system*. If the cardinalities of all minimal paths from $\mathcal{P}$ are equal to k, then $[\mathcal{E}, \mathcal{P}]$ is called a *k-uniform system*. A formation of the system $[\mathcal{E}, \mathcal{P}]$ and its domination $d[\mathcal{E}, \mathcal{P}]$ are defined similarly to the corresponding notions for monotone graphs.

Theorem 2. *The problem of computing the domination is $\#P$-complete in the class of r-cyclic monotone graphs for any fixed integer $r \geq 1$.*

Proof. The following problem (denoted by DS(2)) is known [20] to be $\#P$-complete:

input: a 2-uniform system $[\mathcal{E}, \mathcal{P}]$;

output: the domination $d[\mathcal{E}, \mathcal{P}]$.

Let us take an arbitrary 2-uniform system $[\mathcal{E}, \mathcal{P}]$. We define an (s,t)-graph G as follows:

$$VG = \{s, t, v, w, u_1, \dots, u_{n+r}\},$$
$$DG = \{(w,t), (w,v), (v,w), (s,u_i), (u_i,w) : i = 1, \dots, n+r\}.$$

Also, we set $e_0 = (v,w)$, $e_i = (u_i, w)$, $i = 1, \dots, n+r$. We identify $\mathcal{E}$ with the set of edges $\{e_1, \dots, e_n\}$ and specify the threshold sets of vertices in G as follows: each of the sets $\mathrm{th}(t,G)$, $\mathrm{th}(v,G)$, and $\mathrm{th}(u_i,G)$, $i = 1, \dots, n+r$, consists of the single threshold set (the edge directed to the corresponding vertex)

$$\mathrm{th}(w,G) = \{P_1, \dots, P_m, \ \{e_0, e_i\} : i = n+1, \dots, n+r\}.$$

It is readily verified that exactly r minimal paths of the graph G contain the cycle (w,v), (v,w) (these minimal paths contain the threshold sets $\{e_0, e_i\}$), the remaining m minimal paths are acyclic. In addition, $d(G) = d(w,G) = d[\mathcal{E}^*, \mathcal{P}^*]$, where $\mathcal{E}^* = \{e_0, \dots, e_{n+r}\}$, $\mathcal{P}^* = \mathrm{th}(w,G)$.

Set

$$\mathcal{P}_1^* = \{P_1, \dots, P_m, \ e_{n+1}, \dots, e_{n+r}\}, \qquad \mathcal{P}_2^* = \{P_1, \dots, P_m\}, \qquad \mathcal{E}_0^* = \mathcal{E}^* \setminus \{e_0\}.$$

Then, by the familiar factoring of domination Theorem [6], we have

$$d[\mathcal{E}^*, \mathcal{P}^*] = d[\mathcal{E}_0^*, \mathcal{P}_1^*] - d[\mathcal{E}_0^*, \mathcal{P}_2^*].$$

Since $\mathcal{E}_0^*$ contains elements that do not belong to any minimal path from $\mathcal{P}_2^*$, we have $d[\mathcal{E}_0^*, \mathcal{P}_2^*] = 0$. In addition, $d[\mathcal{E}_0^*, \mathcal{P}_1^*] = (-1)^r d[\mathcal{E}, \mathcal{P}]$, because formations of the system $[\mathcal{E}_0^*, \mathcal{P}_1^*]$ bijectively correspond to the formations of the system $[\mathcal{E}, \mathcal{P}]$ free of the minimal paths $e_{n+1}, \dots, e_{n+r}$. Hence $d(G) = (-1)^r d[\mathcal{E}, \mathcal{P}]$, which establishes the polynomial Turing reducibility of the problem DS(2) to the problem of computing the domination in the class of r-cyclic monotone graphs. This proves Theorem 2. □

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Belorussian Economics University
E-mail address: zhanna_chernyak@usa.net