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About the Influence of a Magnetic Field Gradient on Stability of Ordered Temperature-Electric State in Conductors of Corbino Geometry

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The results of investigation of a temperature field of Corbino sample as a dynamic system are presented. Influence of temperature on generation and distribution of heat is taken into account at non-linearity of a heat removal. For the analysis the integrated Onsager principle of the least energy dissipation in Fourier representation is used. The analysis of a thermal state is executed by a method of representation of temperature as the sum of two functions. One describes some standard condition, and another determines a perturbation as a product of coordinate and exponential time functions. Within the linear approximation the expression for the parameter of stability of perturbed distribution is analysed. The magnetic field and its gradient are taken into account by means of introduction of a magneto-dynamic current correction, a combined magnetothermal correction and a spatial dependence of kinetic coefficients from a magnetic field. It is shown that in conditions of superposition of the external magnetic field and inhomogeneous self-field of carrier drift the value and the direction of a gradient substantially defines the thermal and electric state of a sample.

Key words: electric nonlinearity, magnetic field, Corbino geometry, heat generation, heat removal

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1 Introduction

The nonlinear macroscopic electrical properties of metals wires are widely known via stabilization of current in a regime of voltage supply [1], [2]. The instance of use of this phenomenon is ballast lamp or ballast tube where as known heat regularity is shown through an appearance of domain structure in wire situated in atmosphere of gaseous helium. Domain is a wire section were a temperature and electric field value are largely differ from these parameters averaged on wire length. On this wire section an excessive voltage is fallen so that in some diapason of source voltage supply change the current is constant. For easy realization of ordered temperature and electrical structure the wire length is to be of much higher of its diameter [3] - [5]. Under such conditions the temperature field is homogeneous through cross section.

The observed non-linear electrical phenomena in Corbino disk samples with tendency to voltage stabilization principally differ from current stabilization in ballast lamp on the peculiarities of heat source action [6]. Nevertheless there are definite total signs in that sense that for both cases there is namely a regime close to stabilization of electrical parameters. It reflects the disturbance of macroscopic linear Ohm law. On the other hand the reason of non-linearity in both cases seems to be heat. The direct measurement of local sample temperature stresses this hypothesis [7]. That is a heat field seems to be a structured that as in ballast lamp. Respectively this structurization of thermal field is responsible both for voltage nonlinearity and voltage stabilization under high levels of energy dissipation in Corbino samples. Here some aspects of electrical non linearity of magneto-dynamic nature at steady current flow in regime of current supply are analyzed. The talk is about conditions when the level of energy dissipation is enough to disturb temperature field of sample and respectively the additional mechanisms of systems heat self-organization are turned in.

2 Experimental and analytical procedure

Following data of measurements it is necessary to stress that among measured Corbino samples there were these having different cross sections for radial current [6], [8], [9]. Particularly there was used radial currents cross-section that ensure homogeneity of current density along radial coordinate. Also a samples having reversal dependence of current density on radial coordinate in linear and square law were tested. Respectively for these types of samples local heat source intensity is homogeneous or non-homogeneous in accordance with current density law distribution. Nonlinear electric properties of material in steady field are analyzed via a superposition of thermal and an electrical sources being measured on sample surface with help of potential and temperature probes using an approach of [6]. Stationary thermal state of a cylindrical sample we shall consider using conditions of balance of Joule power dissipating in a volume and removed energy at boiling refrigerant on a surface. The electrical part of analysis consists of macroscopic expression for current-voltage characteristics being an indicator of responsibility of sample behavior to Ohms law.

3 Results and discussion

It was shown that Corbino samples of different axial section profiles have unlike voltage current characteristics [6] - [8]. The reason of existence of different types of characteristics behavior is connected with magnetic field distribution peculiarities and thermal field structure of samples [9], [10]. Monotonous more slight than linear enlarge of voltage with current increase indicates that temperature changes in volume of disk. This changing proceeds simultaneously including both sample central part and a region of outer diameter. Nevertheless the central zone has a slightly higher temperature. The heated region spreads its size with increase of current. For very thin Corbino disk sample the heat field may be taken as almost homogeneous on the reason that small thickness excludes the problems of heat transfer in volume. Any point of sample is close to surface and heat removal through surface is equally effective anywhere.

The peculiarities of heat removal play a main role in appearance of additional heat field after fracture of characteristic. So the heat effects compensate the increase in voltage difference due to current inclusion. It seems that a difference of bigger score between collinear and anti-collinear voltage regime at relatively small magnetic field is a result of less heating of sample than for field of higher order [7] - [11]. The tendency for stabilization is an indicator of heat reorganization. For simple disks with constant height along radius the cross section for radial current is linear function of coordinate along radius. As for former case the heat source is inhomogeneous and local heat generation is reversal function of radius. For a thick disk a heat removal through surface is not effective and thermal conditions are stricter at inner radius. So the excessive dissipated heat spreads along disk radius towards outer diameter. Occurrence of tendency to stabilization is the result of superposition of conditions at gradual change of a thermal condition of the sample corresponding "cold" electric nonlinearity by more high-temperature areas caused by heating processes. Stabilization we connect with the space redistribution of heat field. Redistribution denotes abrupt in time appearance of domain structure.

More clearly regime of stabilization is observable for the case of homogeneous heat source [6].

For sample having hyperbolic profile of axial section the cross section for radial current is constant. It is clear that both a sample surface and heat transfer properties determine the thermal regime in considerable degree.

Under conditions of influence of thermal mechanisms the differential of voltage along a sample radius should be represented in first approximation as the sum of two contributions

$$dU \cong RdI + IdR \qquad (1)$$

Here a resistance is accepted as only temperature function. The first term reflects the Ohm law and the second that characterizes a change of characteristics due to thermal conditions. Total non-monotony of voltage is caused by the more strong reduction of resistance dR with a current growth. In other words at some current levels the reduction of resistance dR suppresses in total potential difference the "current addition" being positive. Note that the differential dR is negative under definite thermal circumstances at large current value.

The behavior of characteristics with taking into account a magnetic field distribution we can explain as behavior that manifests the next dependence on parameters of task $U = IR(I,T)$. Here sample resistance is not only a function of temperature but also depends on current due to cold magneto-nonlinear effect [12]. So a differential of voltage dU is to be expressed as

$$dU = RdI + R'_I dI + IR'_T dT + IR''_{IT} dIdT \qquad (2)$$

Here the first term as in former case is responsible for usual Ohms law under independent resistance, second term describes magnetic nonlinearity, third term is connected with heat nonlinearity and at last the term consisting of multiplication of two differentials is a result of combined influence on resistance of self magnetic field and temperature. In other words R''_{IT} is the magneto-dynamical amendment to a resistance giving both negative and positive contribution to a voltage. Particularly the last term is positive for anti-collinear geometry of current flow because of the temperature part of derivative is negative function. This coefficient is determined by the second derivative on variables and is of higher infinitesimal order. As a result in view of thermal effects an increase of a voltage at collinear geometry is more than that at anticollinear geometry.

Now take into account in-homogeneity of self magnetic field of Hall drift. Another words at anti-collinear geometry the magnetic field in the center of a sample is less than at collinear geometry. This magnetic field distribution reduces heterogeneity of a thermal source for a disk sample with a constant height. Local heating in the center becomes less, resistance is higher and the contribution to a voltage from an inner area grows. As a consequence at such geometry of current flow the probability of transition to condition with occurrence of stabilizing voltage is more favorable than for collinear geometry of current flow. At collinear geometry of current flow the tendency to stabilization is weaker. It is important that for disks having greater thickness the extreme is expressed more strongly, and with magnetic field increase the maximum is displaced in area of smaller values I. More strong expression of extreme for thicker disk sample is consequence of heat field redistribution. Here thermal energy becomes to flow not only along radius but towards disk surface.

As opposed to samples with constant thickness along radius for a disk with hyperbolic axial section a direction of current flow is insignificant for the form of voltage-current characteristics [6], [10]. Here coefficients R''_{IT} and R'_T do not differ strongly for collinear and anti-collinear geometry because of uniformity of a thermal source. Here the only coefficient R'_I results in dynamic nonlinearity.

Sudden jump of voltage U at transition in a condition of stabilization is natural to connect with qualitative change of a temperature condition of system. Some analogy in stabilization principles in a ballast lamp [1], [2] and in a metal Corbino disk [6], [7] is observed. For instance the condition with stabilization of a voltage is asymptotically steady too. It is possible to believe that there is an ordering of a temperature-electric field [9]. Assumptions of an opportunity of occurrence of the domain structure have been confirmed with the help of the temperature gauges mounted on a surface of a sample near to the center [6], [7], [10], [12].

Indirect acknowledgement of a domain structure existing in Corbino-sample is the observable value of voltage that cannot be achieved at heating all the sample as in this case the signal would be much less. In other words the temperature increase of whole sample by one - two degrees would lead to a voltage reduction by tens percent. Thus the facts of voltage stabilization, a spasmodic transition mode, and also an inhomogeneity of temperature field prove the opportunity of existence of a steady domain structure.

The occurrence of the stabilized area in current-voltage characteristic is natural to connect with an opportunity of achievement of a mode of negative differential conductivity [4], [5], [9]:

$$\frac{dE}{dj} = \frac{j}{E}\frac{(\rho q_-)'_T}{(q_-/\rho)'_T} \tag{3}$$

Here j is a current density, E is an electric field intensity, q_- is a heat removal density flux, ρ is a specific resistance. To carry out the analysis of a temperature-electric domain condition of Corbino disk let us consider the functions of heat generation and heat removal being depended on temperature. Here heat removal function is determined with intensity of liquid helium boiling. Bubble boiling is accompanied by thermal pressure when the temperature of a surface exceeds helium temperature on some tenth degree. The transition to film boiling occurs in rather narrow temperature diapason up to shares of degree [13], [14]. Reduction of electric nonlinearity is caused by decrease in a resistance with temperature growth [9]. The dependence of a resistance of thin disk on temperature is submitted in Figure 1. It is visible that the area of a strong magnetic field covers a temperature range from 4.2 K up to 40 K in magnetic field 8.5 T. For weaker fields, for example for $B \cong 1$ T, the Larmor radius equalizes with a

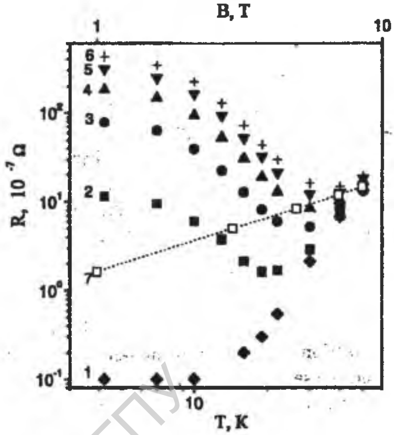

FIG. 1. The resistance of thin disk as a temperature function under magnetic field B_0, T: 0 (1); 1 (2); 3 (3); 5 (4); 7 (5); 8.5 (6). 7 - the resistance in a minimum as a function of magnetic field (top axis).

free length at boiling hydrogen temperature. The resistance in a minimum corresponding to transition from one type of dependence to another is growing function of a magnetic field.

For estimation of a thermal condition of a sample it is necessary to use a dependence of a removed thermal flux on a surface temperature at the thermal conditions being adequate to bubble, film and transitive boiling regimes. For bubble boiling regime the known data [15] are used. For film boiling and for transitive regime the calculation of parameters of the second crisis for a thermal flux q_2 and temperature T_2 is made in accordance with consideration of [13], [14]. So the following expressions are obtained:

$$q_2 \simeq 0.09 r'\rho'' \left[\frac{g\sigma'(\rho'')}{(\rho'+\rho'')^2}\right]^{\frac{1}{4}} \tag{4}$$

$$\Delta T_2 \simeq 0.1 \frac{r'\rho''}{\kappa''}\left[\frac{g(\rho'-\rho'')\eta''}{(\rho'+\rho'')^2}\right]^{\frac{1}{3}}\left[\frac{\sigma'}{g(\rho'-\rho'')}\right]^{\frac{1}{2}} \tag{5}$$

Here r' is a helium evaporation heat, r' and r''

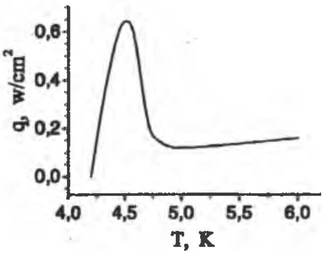

FIG. 2. Helium boiling under normal pressure as a function of surface temperature of aluminum sample.

are the liquid and vapor densities, g is the gravitational acceleration, σ' is a surface tension coefficient, η'' is a dynamic coefficient of vapor viscosity, κ'' is a vapor heat conductivity. The calculated characteristic of a heat-removal by boiling helium as a density of a thermal flux depending on surface temperature with use of parameters of the second boiling crisis (2), (3) is submitted in Figure 2. Such N-type behaviour is connected with lower efficiency of thermal removal by a continuous vapor film in comparison with evaporation of bubbles from a surface in an initial boiling stage. The negative temperature coefficient of a heat transfer in a transitive area is caused by a gradual replacement of a bubble regime by a film one.

On the base of data represented on Figure 1 and Figure 2 it is possible to make qualitative analysis of the processes being realized in Corbino samples. The heat generating function is connected with specific resistance being a temperature function. It is seen that for small currents the temperature is homogeneous, $dE/dj > 0$, and this local relationship is typical for all volume. It corresponds to an increasing branch of characteristic $U(I)$. At this regime there is only one point of intersection of these two functions. At densities j adequate to the large heat generation it is possible to expect $dE/dj < 0$ in all volume, i.e. $dU/dI < 0$. In this mode the characteristic shows the consecutive homogeneous temperature conditions with a negative resistance differential that dominates over current differential. For this mode also a one point of intersection for heat generation and heat removal function takes place. The temperature is to be much higher of liquid helium temperature. This regime may be achieved for a sample of moderate thickness. At last for a sample of small thickness the characteristic does not reach an extreme, and the curve illustrating thermal generation crosses a curve corresponding to heat removal in several points. The number of these points at least is three. This means that some part of sample has more higher temperature and some part of sample has more lower temperature close to temperature of liquid helium.

Note that for helium boiling at homogeneous temperature in sample volume the voltage stabilisation is impossible. Use the equation which connects heat generating function W with voltage differential dU. This equation is $dU \cong 0.5\left[(dW)_{R=Const} + 2(dW)_{I=Const}\right] I^{-1}$ (here as mentioned early $W = q_+$ is an integrated value of a thermal generation depending on T). As follows from the given equation the existence of stabilised area $dU = 0$ with the homogeneous distribution of temperature is possible only when the integrated functions of a thermal generation $'q_+$ and a heat removal $'q_-$ at changing current can lead to the definite set of conditions. In this case an initial condition having a thermal generation function RI^2 passes with the current increase in state $R(I + dI)^2$. After an establishment of thermal balance it passes in a condition with $'q_+ = (R + dR)(I + dI)^2$. The common increase of thermal generation function is $\Delta' q_+ = 2RIdI + I^2dR$, and the increase of a voltage corresponds $dU = RdI + IdR$. Thus dU is equal zero if $0.5(\Delta' q_+)_{R=Const} = -(I^2dR)_{I=Const}$. Following character of inclinations of boiling and heat-removal curves submitted in Figure 1 and Figure 2 it is seen that the bubble boiling regime results in positive dU through all surface. Respectively the film boiling regime leads to negative dU.

This approach allows to interpret all experimental dependences. As the kind of function

$\Delta' q_+$ in conditions of experiment is too far from symmetric, the observable stabilization of integral voltage can arise due to combination of areas with positive and negative differential conductivity dE/dj. The denominator in Eq. (3) is positive outside of a transitive area. The numerator is positive in bubble branch and negative in film one. In intermediate area the temperature dependence of magnetoresistance $\rho \cong AT^n$ has an exponent $n \cong -0.45$. In view of sharp reduction q_- the sign of both numerator and denominator is negative. In result a system after achievement of a threshold condition can pass into a new mode when some part of a sample has T close to 4.2 K. In this time another part is warmed up to a level determined by a point of crossing of heat generation and heat removal curves in film boiling area.

4 Concluding remarks

On total influence on heat generation a negativity of temperature derivative of heat source for regime of current supply in Corbino sample is similar to negativity of temperature derivative of heat source for regime of voltage supply in ballast lamp. Respectively stationary state with ordered temperature structure being that of domain type is result of combination of heat generation function and nonlinear heat removal function. These functions intersect in some points indicating the presence of sample parts having different temperatures. The differential conductivity in these parts of sample is both positive and negative that. For anti-collinear geometry of current flow the self-magnetic field gradient is directed to outer diameter of disk sample. This obstacle decreases heterogeneity of heat source if other measures for heat generation homogeneity are not accepted. Respectively at anti-collinear geometry the conditions for establishing of ordered temperature field are more favorable and there are more chances for observing a stabilization regime.

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